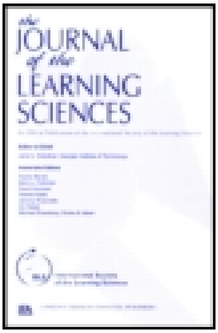

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# Exploring the Use of Conceptual Metaphors in Solving Problems on Entropy

Fredrik Jeppsson [a] , Jesper Haglund [a] , Tamer G. Amin [b] & Helge Strömdahl [a]

[a] The Swedish National Graduate School in Science and Technology Education, Linköping University

[b] Department of Education, Lebanese American University

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Fredrik Jeppsson and Jesper Haglund
*The Swedish National Graduate School in Science and Technology Education*
*Linköping University*

Tamer G. Amin
*Department of Education*
*Lebanese American University*

Helge Strömdahl
*The Swedish National Graduate School in Science and Technology Education*
*Linköping University*

A growing body of research has examined the experiential grounding of scientific thought and the role of experiential intuitive knowledge in science learning. Meanwhile, research in cognitive linguistics has identified many *conceptual metaphors* (CMs), metaphorical mappings between abstract concepts and experiential source domains, implicit in everyday and scientific language. However, the contributions of CMs to scientific understanding and reasoning are still not clear. This study explores the roles that CMs play in scientific problem-solving through a detailed analysis of two physical chemistry PhD students solving problems on entropy. We report evidence in support of three claims: a range of CMs are used in problem-solving enabling flexible, experiential construals of abstract scientific concepts; CMs are coordinated with one another and other resources supporting the alignment of qualitative and quantitative reasoning; use of CMs grounds abstract reasoning in a "narrative" discourse incorporating conceptions of paths, agents, and movement. We conclude that CMs should be added to the set of intuitive resources others have suggested contribute to expertise in science. This proposal is consistent with two assumptions: that cognition is embodied and that internal

Correspondence should be addressed to Fredrik Jeppsson, the Swedish National Graduate School in Science and Technology Education, ISV, Linköping University, 601 74 Norrköping, Sweden. E-mail: fredrik.jeppsson@liu.se

cognitive structures and processes interact with semiotic systems. The implications of the findings for learning and instruction are discussed.



In this article, we analyze a problem-solving session in which two PhD students together solved problems on entropy. The goal of our analysis was to understand how conceptual metaphors (CMs) contribute to expert scientific problem solving. As we explain, a CM involves an abstract conceptual (target) domain being construed in terms of a more concrete conceptual (source) domain, and it is often reflected in language use (Lakoff & Johnson, 1980, 1999). Our hypothesis is that source domains that incorporate experiential conceptions of objects, substances, locations, paths, and others are used as resources in scientific problem solving. Characterizing the roles of these conceptions in problem solving provides clues about their role in the learning process, which has implications for pedagogy. Thus, we seek to add the source domains of CMs to the list of cognitive resources previously proposed in the literature as being available to the novice and contribute to expert understanding and reasoning (e.g., Amin, 2009; Clement, 2009; diSessa, 1993b; Gupta, Hammer, & Redish, 2010; Hammer & Elby, 2003; Sherin, 2001). Three previous proposals in the literature are particularly similar to our own: diSessa's (1993b) claim that phenomenological primitives (p-prims) contribute to scientific understanding and reasoning, the claim that novices' and experts' ontological construals of concepts are flexible (Amin, 2009; diSessa, 1993b; Gupta et al., 2010), and Sherin's (2001, 2006) suggestion that understanding physics equations relies on symbolic forms. As we clarify, our hypotheses are consistent with these accounts but complement them by examining the role that pervasive metaphorical mappings implicit in language use play in scientific problem solving. In the next three sections, we situate our study with respect to prior research and describe the framework of CM that guides our investigation.

## NOVICE AND EXPERT SCIENTIFIC UNDERSTANDING AND PROBLEM SOLVING

An initial phase of inquiry into expertise in science and its acquisition sought to identify differences between expert and novice reasoning (Bhaskar & Simon, 1977; Chi, Feltovich, & Glaser, 1981; de Kleer, 1977; Larkin, 1983; Larkin, McDermott, Simon, & Simon, 1980). This research made some valuable contributions. Novices were found to represent and sort physics problems in terms of surface features (the presence of pulleys, inclined planes, etc.), whereas experts used representations of idealized objects, scientific concepts, and principles (Chi et al., 1981). It was found that experts, but not novices, possess elaborate domain-specific schemata, including idealized qualitative representations of common physical situations and scientific concepts and principles (Bhaskar & Simon, 1977; Chi et al., 1981; Larkin, 1983; Larkin et al., 1980). It was suggested that novices reason backward from unknowns, accessing and stacking needed equations until a solution is found, whereas experts tend to reason forward from givens to unknowns (Larkin et al., 1980)—although this generalization has since been questioned by Priest and Lindsay (1992). Moreover, it was suggested that scientific concepts classified by experts as constraint-based processes are often classified incorrectly by novices in terms of a material substance ontological category (Chi & Slotta, 1993).

This research was conducted within the computational paradigm dominating cognitive science at the time. Among the key goals of this work was to characterize expert and novice problem solving in terms of broad types of representations and computational processes, and with sufficient precision to enable the construction of computational models that could mimic observed performance (e.g., Larkin, 1983; Larkin et al., 1980). Representations were contrasted in terms of broadly distinct types of informational content (e.g., surface features vs. scientific concepts and principles), but the format of the representation was not under discussion. Computational models were implemented in terms of (language-like) propositional representations and processes (e.g., list structures to represent physical objects and their properties and production rules to generate new representations).

Recently, there has been growing interest in understanding continuity between novice and expert reasoning (e.g., Amin, 2009; Clement, 2009; diSessa, 1993b; Gupta et al., 2010; Hammer & Elby, 2003; Sherin, 2001). Three key themes run through this literature: (a) Rather than contrasting the knowledge representations of novices and experts in terms of broad types of informational content, researchers are identifying the many sense-making resources available to the novice that could play a role in expert understanding and reasoning; (b) rather than assuming language-like propositional representations and computational processes, there is a growing commitment to embodiment, in which abstract knowledge and reasoning are viewed as grounded in early bodily based interactions with the world; (c) it is recognized that external semiotic systems such as language, equations, images, diagrams, graphs, physical artifacts, and gestures interact with internal cognitive resources to support understanding and reasoning.

## Theme 1: Everyday Sense-Making Resources as Contributors to Expertise in Science

The first theme is well illustrated by "knowledge-in-pieces" research that has sought to identify the numerous everyday sense-making resources that contribute to expert understanding and reasoning (e.g., diSessa, 1993b; Hammer & Elby, 2003; Hammer, Elby, Scherr, & Redish, 2005; Redish, 2004; Sherin, 2001; Wittmann, 2002). For example, diSessa (1993b) has proposed that the everyday "sense of mechanism" consists of a large number of knowledge elements that he calls p-prims that allow people to explain why objects move or remain stationary, to generate predictions about interacting objects, and so on. An example is Ohm's p-prim, a simple yet structured schema that incorporates the notion of an agent that acts on some resisting entity to achieve a result. It can support inferences such as "more effort can produce more result" or "more effort is required with greater resistance." diSessa has suggested that individuals possess a very large number of such intuitive knowledge structures, such as *force as mover, opposing forces,* and *balancing,* among many others. If misapplied, p-prims will produce unscientific conclusions about physical situations. However, they can contribute to scientific reasoning if appropriately applied. For example, Sengupta and Wilensky (2009) argued that misconceptions regarding electric circuits are often due to misapplication of p-prims at the macroscopic level of electric current and voltage but that learners more easily apply p-prims in a productive way at the microscopic level with a focus on free electrons. Similarly, Sherin (2001) analyzed the use and conceptualization of equations in the context of scientific problem solving. He documented the use of intuitive conceptual schemata that are similar to, and in some cases *are,* p-prims that are used to interpret patterns of symbols in equations. For example, the generic pattern of symbols, or "symbol template," consisting of two terms separated by an equals sign ($\square = \square$), can be interpreted in terms of the conceptual schema of two balancing influences (which is the p-prim *balancing*). Together a conceptual schema and symbol template constitute a "symbolic form." The expert problem-solver will draw on many symbolic forms that give meaning to equations and support connections between qualitative and quantitative reasoning in physics. It is important to highlight here that the conceptual schema, the meaning pole of a symbolic form, is often a resource readily available to the novice learner.

Other kinds of useful knowledge elements available to the novice have been identified and their organization and application have been examined. Clement, Brown and Zietsman (1989) described "anchoring intuitions" (e.g., a spring applies an upward force on an object placed upon it) that can support scientific understanding (e.g., a normal force acts on an object resting on a surface). Hammer and Elby (2003) identified "epistemological resources," useful everyday intuitions about the nature of knowledge. Moreover, research has sought to understand how multiple resources are applied and collectively constitute expertise. diSessa and colleagues (diSessa, 2002; diSessa & Sherin, 1998; diSessa & Wagner, 2005) have developed an account of concepts as complex knowledge systems ("coordination classes"), in which diverse resources are coordinated to realize general and consistent scientific understanding and reasoning. From this perspective, transitioning from novice to expert involves developing increasingly organized knowledge systems through the incorporation, displacement, and coordination of elements. In related work, Hammer et al. (2005) proposed the idea of "framing," a broad interpretation of the context of reasoning that affects which knowledge resources are drawn on in any given situation.

### Theme 2: The Embodiment Assumption

The embodiment assumption is that knowledge and reasoning are grounded in early bodily based interactions with the world. diSessa (1993b) suggested that p-prims are small knowledge structures that are generalized schemata derived from sensorimotor experience (i.e., pushing, lifting, carrying, and experiences of balance and imbalance are the experiential sources of p-prims). It is in this sense that p-prims are embodied knowledge structures. That is, the assumption is that scientific concepts formulated in terms of formal symbols and reasoning involving the formal manipulation of these symbols via logical deduction rely on sensorimotor schemata and the inferential capacities that these schemata support.

This embodiment assumption is reflected in one side of a debate on the role of ontological categorization, initiated in 1993 (Chi & Slotta, 1993; diSessa, 1993a, 1993b) and continuing recently in the *Journal of the Learning Sciences* (Chi, 2005; Gupta et al., 2010; Hammer, Gupta, & Redish, 2011; Slotta, 2011). As mentioned previously, it has been proposed that science learning involves an ontological reclassification of concepts from a material substance to a constraint-based interaction (or emergent process) category (Chi, 2005; Chi & Slotta, 1993; Chi, Slotta, & De Leeuw, 1994; Slotta & Chi, 2006). However, Chi and her colleagues have been criticized for inappropriately viewing the shift from novice to expert classification of concepts as a shift from one static, stable ontological category to another (Gupta et al., 2010; Hammer et al., 2011). That is, Gupta and colleagues argue that both learners and experts construe concepts within ontological categories in a considerably more flexible way, straddling ontological categories. Gupta et al. (2010) provided evidence that experts *and* learners talk and reason about emergent processes *as if they were* material substances. Although the debate between the ontological shift and flexible ontology proponents continues, there is increasing plausibility to the claim that expert scientists' formal conceptual understanding draws on concrete notions of material substance.

Further support for the embodiment assumption comes from other lines of research. Historical case studies of scientists' creative reasoning have documented the importance of model-based reasoning, involving non-formal knowledge structures and processes such as sensorimotor schemas, imagistic simulation, and analogical reasoning (e.g., Nersessian, 2008; Tweney, 1992). In addition, Dunbar (1997) has catalogued a variety of uses of analogical reasoning in scientific laboratory meetings, from dealing with details of experimental design to forming explanatory hypotheses. Moreover, based on analyses of think-aloud protocols of scientists solving problems outside of their areas of expertise, Clement (2009) documented how insights are arrived at through the activation of analogies and model construction and revision grounded in the use of perceptual motor schemas and imagistic simulation. That formal scientific understanding and reasoning are grounded in bodily based experiences suggests that pedagogical strategies that recruit such experiences will support student learning. Indeed, Clement's research has shown how the same non-formal structures and processes revealed in scientists' thinking can be used by students to achieve meaningful understanding of challenging scientific concepts. Moreover, embodied modeling teaching approaches have proved successful. For example, Wilensky and Reisman (2006) used an "agent-based" approach to computer-based modeling of changes in biological populations, allowing students to take the perspective of a single organism in order to ground their understanding of the more abstract aggregate level. In addition, a recent special issue of the *Journal of the Learning Sciences* was devoted to embodiment in teaching and learning mathematics (Hall & Nemirovsky, 2012).

## Theme 3: The Interaction Between Semiotic Systems and Internal Cognitive Resources

The interaction between external semiotic systems—such as language, equations, images and diagrams, graphs, physical artifacts, and gestures—and internal cognitive resources has been examined in expert understanding and reasoning as well as in learners on the path to expertise. Important contributions to understanding this interaction in expertise have come from historians and philosophers of science (Giere, 2002; Nersessian, 2008) drawing on basic cognitive science research (Clark, 1997; Hutchins, 2005; Rumelhart, Smolensky, McClelland, & Hinton, 1986). Giere (2002) has suggested that it is unlikely that scientific models, given their complexity, can be represented solely in the form of internal mental models. He argued that internal mental representational capacities need to be augmented by external semiotic tools. Nersessian (2008) has argued that cases of conceptual change in the history of science and creative insight in analyses of a contemporary scientist's reasoning were supported by the use of visual representations of various kinds. In learning sciences research, Sherin's (2001, 2006) proposal (reviewed in the previous section) that internal conceptual schemata are associated with symbol templates and recruited in scientific problem solving, giving meaning to equations, addresses the interaction between internal cognitive resources and the semiotic system of algebraic equations.

There is little research on the interaction between language and internal cognitive processes in scientific expertise (Amin, 2012). However, Ochs, Gonzales, and Jacoby (1996) identified a form of scientific discourse that they proposed is particularly oriented to thinking through problems. They situated this discourse as intermediate between subjective, "physicist-centered" discourse and objective, "physics-centered" discourse. In these two forms, thematic agents are either the physicists designing experiments and experiencing phenomena or the physical entities and processes being investigated, respectively. Distinct from both forms is a form that blends both of them using pronouns with "indeterminate" reference. An example of this is when a condensed matter scientist studied by Ochs et al. says, "When I come down I'm in the domain state" while discussing a phase change of a system in the presence of a graphical representation of phases of the system as a function of temperature and magnetic field. Ochs et al. suggested that this "referential ambiguity is a necessary poetics of mundane scientific problem solving in that by using indeterminate constructions as a linguistic heuristic, scientists constitute an empathy with entities they are struggling to understand" (p. 348). They argued that the significance of this form of discourse is reflected in famous accounts of scientific insight involving reports of imagery in which the scientist adopts the perspective of a physical entity being investigated. Moreover, they noted that this form of discourse appeared most frequently in the lab discussions analyzed when the scientists engaged in a "thinking through process." But what cognitive contribution is made to the reasoning by this discursive form was not addressed. In an extended theoretical formulation of the knowledge-in-pieces view, diSessa (1993b) suggested that language and algebraic formulations of physical laws can be seen as playing the role of collecting and guiding the application of collections of p-prims. Sherin's (2001, 2006) work on symbolic forms is an empirical investigation of this idea in the case of algebraic equations. However, the relationship between language and p-prims in scientific expertise has not been empirically explored.

Interaction between external semiotic systems and internal cognitive processes has also been examined in science learners (Roth & Lawless, 2002; Sherin, 2006; Svensson, Anderberg, Alvegård, & Johansson, 2009). A large body of research has documented the role of visual representations in guiding analogy-induced conceptual change during science learning (e.g., Clement, 2009; Wiser & Amin, 2002). Roth and Lawless (2002) suggested that gestures are used by learners to embody conceptual entities and, along with observed objects and events, support linguistic communication. They pointed out that this use of gesture frequently precedes the formulation of the same understanding in language, thus acting as a bridge between experiences of the observed world and more formal conceptual understanding. Sherin (2006) examined the use of symbolic forms in scientific problem solving, also from a learning perspective. He suggested that the use of p-prims is refined in the context of problem solving itself, where the manipulation of equations can increase the priority of a useful p-prim. Moreover, as mentioned previously, p-prims are recruited as part of symbolic forms and constitute the understanding of an equation itself. Sherin (2006) suggested that, in order to play the role required of them in a symbolic form, some p-prims will be modified, a process often involving the "washing out of physical meaning" (p. 552). For example, the p-prim "more physical effort implies more result" becomes simply "more is more," which then plays the role of the conceptual schema in a proportionality symbolic form.

Some research has examined the relationship between language and everyday intuitions (Amin, 2009; diSessa, 1993b, 1998; diSessa, Elby, & Hammer, 2003; diSessa & Sherin, 1998; Levrini & diSessa, 2008; Svensson et al., 2009). Examination of learners' language use in early attempts to reason scientifically has often revealed substantial contextual instability in the use of terms (diSessa et al., 2003; Svensson et al., 2009). Novices not only apply experience-based intuitions inappropriately but often use a variety of different terms to express the same idea, or use a single term for a number of different ideas (Svensson et al., 2009). diSessa et al. (2003) suggested that this instability in learners' use of language reflects the absence of a metalinguistic commitment to generality and consistency in the use of scientific terms. In addition, in the context of a survey of different types of resources that can help to characterize naive and expert understanding diSessa (1998) mentioned nominal facts and narratives. Nominal facts are learned factual statements that derive from novices' brief exposure to science, but, according to diSessa (1998), they "frequently have so little meaning for students that the meaning and implications of the facts are haphazard consequences of the situation in which students try to use them" (p. 719). For example, "heat is transferred from a hot object to a cold object" may be recalled as a fixed phrase without conveying any meaning. Narratives, like nominal facts, are superficial verbal resources that have been memorized with little understanding. To diSessa (1998), narratives are easily learned by physics students. However, once a commitment to meaningful, coherent, consistent language use is in place, language can impose a kind of top-down coherence in the appropriate application of p-prims, an early theoretical proposal made by diSessa (1993b). Levrini and diSessa (2008) have shown empirically how definitions introduced by a teacher supported students in applying their understanding of proper time across a wider range of contexts. However, Levrini and diSessa did not document how definitions guide the application of particular p-prims. Thus, absent from this work is the characterization of how language interacts with specific intuitive knowledge structures to support understanding and reasoning in a way that resembles the work on gesture (Roth & Lawless, 2002) and equations (Sherin, 2001, 2006) reviewed previously.

In the study reported in this article, we seek to contribute to the three themes just reviewed by investigating the role of CMs in scientific problem solving on entropy. In the next two sections, we introduce the area of thermodynamics and the concept of entropy addressed in the present study and the theoretical perspective of CM we draw from in our analysis, highlighting its contribution to each theme.

## THERMODYNAMICS AND THE CONCEPT OF ENTROPY

Entropy and the second law of thermodynamics are linked to understanding of the direction of spontaneous processes. For instance, reversing the process of a glass of milk being spilled over the floor is highly unlikely. Introducing the concept of entropy is a way to operationalize this physical phenomenon; irreversible change is always accompanied by increasing entropy. Unlike irreversible processes, a reversible process can be driven in the opposite direction without the dispersal of energy, and hence the entropy is constant. In statistical mechanics, the equation $S = k_B \ln \Omega$ states that the entropy $S$ is proportional to the natural logarithm of the number of microstates of a system $\Omega$ (i.e., the number of microscopic configurations of the system).

Baierlein (1994) suggested that there are generally two approaches to the field of thermal phenomena. On the one hand, a macroscopic approach can be adopted that focuses on properties of cyclic processes involving heat and work. On the other hand, a microscopic statistical mechanics approach involves the introduction of a system–particle model and microstates. It is a challenge for learners to create the appropriate links between these two levels of description. In line with Baierlein, Slotta and Chi (2006) made the more general claim that students have difficulty understanding *emergent processes,* in which manifestations at a macroscopic level correspond not to the behavior of individual particles but to their statistical, average behavior, such as energy transfer by heat conduction or the motion of electrons in an electric circuit. Empirical studies in science education have identified the concept of entropy and the second law of thermodynamics as challenging for learners (e.g., Brosseau & Viard, 1992; Sözbilir & Bennett, 2007). In addition, specific challenges with instructional metaphors have been noted. Brosseau and Viard (1992) conducted an interview study among university physics students examining the conceptual understanding of entropy. They found that only 1 out of 10 students grasped the idea that entropy is constant during adiabatic (when no heat is exchanged with the surroundings) reversible expansion of an ideal gas. Seven of the others argued that because the volume increases, so does the "disorder" and, hence, the entropy. By applying the disorder metaphor in this way, the students incorporated only the contribution of spatial configuration to the entropy and failed to acknowledge the energy contribution, which also needs to be considered when characterizing microstates. Sözbilir and Bennett (2007) provided similar empirical evidence demonstrating that by using the metaphor "entropy is disorder" students may reach erroneous conclusions during problem solving in thermodynamics. Within science education research there is an ongoing debate over whether to use the "entropy is disorder" metaphor. Lambert (2002) argued that the disorder metaphor misleads students, as just discussed.

Debate about the value of a variety of different instructional metaphors for teaching entropy and the second law of thermodynamics can be found in the science education literature (see Jeppsson, Haglund, & Strömdahl, 2011, for an in-depth discussion). This debate is concerned with the explicit use of metaphor in instruction. As we discuss, research on CM has revealed extensive and systematic use of metaphorical mappings implicit in apparently literal and unremarkable language use. This implicit use of metaphor is found in everyday as well as scientific language, including that dealing with thermodynamics and the concept of entropy.

## CONCEPTUAL METAPHOR: LANGUAGE AND GROUNDING UNDERSTANDING OF ABSTRACT CONCEPTS

Cognitive linguistic research on CM has documented patterns in everyday language that reflect systematic mappings between concrete and abstract conceptual domains (Lakoff & Johnson, 1980, 1999). *Concrete* domains are those conceptual schemata that derive directly from the sensorimotor experience of interacting with the physical and social worlds, whereas *abstract* domains are those schemata for which no such experiences can be identified. CM involves the use of metaphorical mapping to recruit experience-based schemata to ground the understanding of abstract domains. Mundane expressions such as "I'm *in* love," "I *fell into* a depression," and "He *pulled me out of* my slump" are viewed as instances of metaphorical language that reflect a systematic set of mappings between conceptual domains in which states are construed as locations, changes of state are construed as movement into or out of a location, and caused changes are represented as forced movement. Lakoff and Johnson (1999) referred to this set of mappings as the Location Event Structure CM. Love Is A Journey ("We'll just have to *go* our *separate ways*") and Time Passing Is Motion Of An Object ("Summer has finally *arrived*") are just two of the vast collections of other CMs that have been identified in the literature (see Kövecses, 2010, for a comprehensive overview). Of particular interest here is that many CMs involve mappings from concrete knowledge structures incorporating notions of object, space, movement, and force to abstract concepts such as time, cause, depression, and love for which no (or little) experience-based schematic structure can be described.

This phenomenon extends to the language of science and mathematics. Amin (2009) identified many CMs implicit in the scientific use of the term *energy.* Energy was construed as a substance transferred from one entity to another or as a whole object composed of parts, forms of energy were construed as containers, and others. Moreover, Amin, Jeppsson, Haglund, and Strömdahl (in press) identified a range of CMs implicit in the language of textbooks dealing with the concept of entropy and the second law of thermodynamics. Many of the CMs used at the macroscopic level were applications of the Location Event Structure CM, such as Thermodynamic States Are Locations, as in "the three phases are present *in* thermodynamic equilibrium" (Bowley & Sánchez, 1999, p. 34); Changes in Thermodynamic States Are Movements, as in "when the process *goes from* state 1 . . . *to* state 2" (Zumdahl, 1998, p. 398), and Spontaneous Change Is Directed Movement, as in "many thermodynamic processes proceed naturally *in one direction*" (Young, Freedman, & Sears, 2003, p. 754; emphasis added to all three quotations). An example of the relatively few CMs at the microscopic level is Change Of State Is Movement Into/Out Of A Location, as in "the molecules *go into* solution independently of each other" (Bowley & Sánchez, 1999, p. 70, emphasis added). CMs were also used to construe the relationship between levels, such as Relating Ideas At Different Levels Is To Connect Them, as in "Z acts as a *bridge connecting* the microscopic energy states of the system *to* free energy" (Bowley & Sánchez, 1999, p. 97, emphasis added). The use of CMs in scientific language has been described for a number of other topics, including quantum mechanics (Brookes & Etkina, 2007), the theory of natural selection (Al-Zahrani, 2008), biochemical processes (Semino, 2008), and others (T. Brown, 2003). Moreover, CMs in mathematics have been richly described by Lakoff and Núñez (1997, 2000). In relation to the role of embodied cognition in mathematics education, Williams (2012) provided an image schema analysis of the resources elementary students use in learning to read the clock. He argued that image schemas can be used in a productive way but can also be misapplied by overextension. In particular, by use of the "proximity" of the short hand to the closest whole hour, "a quarter to four" is easily misread as "4:45."

The question that emerges from such CM analyses is the extent to which the CMs are resources for *understanding* and *reasoning*. Lakoff and Johnson (1980, 1999) have assumed that CMs are part of the human *conceptual* system, underlie language comprehension, and support reasoning. Support for this view has come from psycholinguistic research on the comprehension of everyday metaphorical language (see Gibbs, 2005, for a review). Moreover, careful analysis of published research on attention led Fernandez-Duque and Johnson (1999, 2002) to propose that CMs have guided hypothesis formation and research design. However, the view that CMs contribute to understanding and reasoning has not been without its critics (e.g., Haser, 2005; McGlone, 2007; Murphy, 1996). Critics have questioned the strong claim that the source domains of CMs entirely constitute the representational content of target concepts, the weaker claim that they only partially structure target concepts, as well as the weakest claim that it is only the *process* of understanding of metaphorical *expressions* that involves the retrieval of CMs. Rebuttals in response to these criticisms have been formulated (Gibbs, 1996; Gibbs, Costa Lima, & Francozo, 2004). In particular, in the context of discussing research on embodiment in mathematics education, Núñez (2012) and Stevens (2012) have pointed out the need for diverse forms of empirical evidence for claims made by cognitive linguists regarding thinking and learning. A detailed discussion of this debate is beyond the scope of this review, but both supporters and critics agree that there has been an overreliance on isolated sentences as a source of data, and they call for more empirical research that draws on a wider range of methods.

To our knowledge there is no empirical research documenting the role CMs play in *scientific* reasoning and problem solving. Thus, one goal of this study is to provide evidence that the source domains of CMs are activated as cognitive resources during scientific problem solving. To provide such evidence is to contribute to the first theme (discussed in the previous section) by proposing another type of intuitive knowledge structure that can be assumed to be available to the learner and that plays a role in scientific expertise. That CMs would be a *new* kind of intuitive resource can be clarified by examining how CMs relate to the two other themes.

First, embodiment is also an assumption of the CM framework. It is assumed that schemata derived from generalization over sensorimotor experience ground understanding of abstract concepts via metaphor (Johnson, 1987; Lakoff, 1990). Many source domains are assumed to be structured in terms of "image schemas" intermediate in degree of abstraction between propositional linguistic representations and specific images. Johnson (1987) explained the nature and importance of image schemas as follows:

> The view I am proposing is this: in order for us to have meaningful, connected experiences that we can comprehend and reason about, there must be a pattern and order to our actions, perceptions, and conceptions. *A schema is a recurrent pattern, shape, and regularity in, or of, these ongoing ordering activities.* These patterns emerge as meaningful structures for us chiefly at the level of our bodily movements through space, our manipulation of objects, and our perceptual interactions. (p. 29)

That is, image schemas are the same kind of knowledge structure as p-prims: Both are structured schematizations of sensorimotor experience. diSessa (1993b, p. 122) noted this similarity between the two constructs but emphasized that his interest was in identifying intuitive structures that implicate notions of force and motion so as to understand the contribution of everyday sense-making of relevance to understanding the domain of mechanics. That is, his focus was on the application of p-prims *within* the domain of force and motion. diSessa pointed out that, in contrast, Johnson's interest in image schemas was *across* domains; the horizontal metaphorical projection between domains.[1] More recently, diSessa (2000) has gone on to point out that the p-prims that contribute to the intuitive "sense of mechanism" are also recruited in reasoning about the social world. This is metaphorical projection, although diSessa does not discuss this phenomenon in terms of metaphor explicitly. Therefore, the difference between diSessa's proposal that p-prims are resources for scientific reasoning in mechanics and our suggestion here that source domains of CMs are used in solving problems on entropy is our addition of *metaphor* as a mechanism for projecting image-schematic structures to abstract domains.

Second, the mechanism of metaphorical projection is closely tied to language. The source domains of CMs are reflected in linguistic elements: Prepositions such as *from, into, along,* and *through* mark spatial and motion schemas; action verbs such as *push, bring, give, pick up, add,* and *raise* express notions of force, motion,

[1] Roschelle (1992) has pointed out that the use of p-prims within the domain of mechanics can also be given a metaphorical reading. For instance, the interpretation of an acceleration vector "pulling" a velocity vector makes use of a p-prim (force as a mover) in a metaphorical way; there is actually no entity performing a pulling action.

and exchange of possession; adjectives such as *large* and *small* express size; and many others. Such linguistic elements can be used to invite the metaphorical construal of one conceptual domain in terms of another. Everyday and scientific language alike have evolved to include conventional patterns of metaphorical projection associated with uses of linguistic elements. This is analogous to Sherin's (2001) notion of a symbolic form. Just as a symbol template comes to be associated with a p-prim or other conceptual schema and supports expert understanding, particular uses of linguistic elements in the context of talking about scientific topics are associated with a metaphorical mapping between a source and target domain. In sum, p-prims, source domains of CMs, and Sherin's conceptual schemas are the same kind of knowledge structure—they are all image schemas—but are evoked differently. It is possibly useful to think of image schemas as the larger set, with the others as subsets: If an image schema is systematically projected to another conceptual domain it is viewed as a source domain of a CM (often marked linguistically); if an image schema is recruited to provide meaning for a pattern of symbols in an equation it becomes the conceptual schema of symbolic form; if an image schema is not conventionally associated with any external representational structure it is a p-prim.

In this article, we ask the following: What image-schematic source domains are metaphorically projected onto abstract conceptual target domains in advanced scientific problem solving, and how do these metaphors contribute to problem solving? To answer this question we present a detailed analysis of two PhD students thinking aloud together as they solve a series of problems on entropy. The main focus of the analysis is on identifying the CMs used and characterizing the roles of their source domains in the problem solving. How these CMs were used in conjunction with other resources (e.g., symbolic forms) is also examined. We have chosen the concept of entropy because it is a topic that has been found to be pedagogically challenging for learners in large part because of the abstract nature of the concept (e.g., Christensen, Meltzer, & Ogilvie, 2009; Cochran & Heron, 2006; Jeppsson et al., 2011). After describing the research methodology used and presenting the results, we discuss the implications of this research for learning and instruction in this domain and in science more generally.

## METHOD

### Overview of the Study Design

The empirical data that served as the basis for the analysis reported in this article were collected as part of a larger project on problem solving in the field of thermodynamics with a particular focus on entropy. That larger project sought to identify differences in understanding and reasoning in this field between undergraduate students and PhD students specializing in physical chemistry. The latter group was assumed to possess an adequate knowledge of the relevant theories and problem-solving strategies. The problems were designed so that different problem-solving approaches—including microscopic and macroscopic perspectives—would be suitable for different problems. In addition, the exercises were designed to expose shortcomings in the use of the instructional metaphor "entropy is disorder" and to probe the understanding of the extensive character of entropy. In this article, we report on the analysis of the verbatim transcript of the problem-solving session of two PhD students. The Metaphor Identification Procedure (MIP; Pragglejaz Group, 2007) guided the identification of CMs in the transcript, and the procedure for analyzing qualitative data recommended by Chi (1997) was used to investigate how CMs were used to support scientific reasoning.

## Participants

The participants were two Swedish doctoral students (henceforth, D1 and D2) acquainted with thermodynamics at the postgraduate level. They participated voluntarily in the study with no compensation. At the time of the study, D1 was in his fifth year as a doctoral student in physical chemistry and was studying the interaction energies between surfaces in solvents. He had completed an MSc degree in physics and electrical engineering and had taken graduate courses on statistical thermodynamics and physical chemistry. D2 had been a doctoral student in physical chemistry for about 4 years and was studying charged molecule clusters within the framework of his thesis. He had completed an MSc degree in chemical engineering with engineering physics; had professional experience with thermodynamics at an advanced level in his profession; and had taken graduate courses on statistical thermodynamics, physical chemistry, and statistical physics for nanoclusters and thermodynamics.

## Problems

The participants were given three written problems in English as presented here (see the Appendix, which provides detailed solutions to all problems as a guide for the reader). The diagrams accompanying the problems are presented in Figures 1–3.

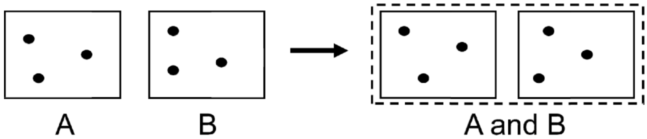


FIGURE 1 Diagram of two systems considered together in Problem 2a.

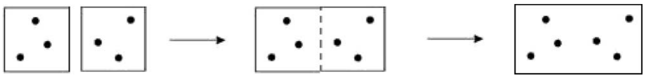

FIGURE 2 Diagram of two combined systems considered in Problem 2b.

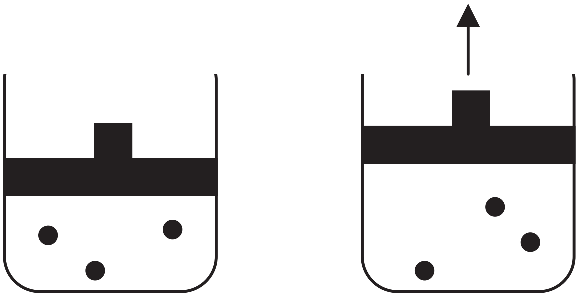

FIGURE 3 Diagram of adiabatic expansion in Problem 3.



Problem 1 considered the situation of putting a beaker of water in a freezer:

> A beaker contains water at temperature 0 °C and is put in a room of air at a constant temperature of –10 °C, so that a layer of ice forms on top of the liquid water. Describe what drives forward the process of freezing the water.

In this problem, we were expecting an explanation of how the water could freeze spontaneously in spite of the apparent increase in the degree of order considered from a spatial, visual point of view. As mentioned previously, the disorder metaphor for entropy has been recognized as potentially misleading for students (e.g., Lambert, 2002). An appropriate explanation could be based on the entropy increase due to released energy in the phase change from liquid water to solid ice being larger than the entropy decrease due to the restriction in spatial configuration at temperatures below 0 °C.

Problem 2 (see Figures 1 and 2 for diagrams considered in Problem 2a and 2b, respectively) considered the entropy of combined systems:

> Two identical isolated systems A and B contain a monatomic ideal gas, each one with number of particles N, volume V, temperature T and entropy S.
>
> (a) When the two systems are considered together as 'A and B,' the temperature is T, the volume is 2V and the number of particles is 2N. What is the entropy of the combined system 'A and B'?
>
> (b) Assume that the two systems are put in contact, so that particles and energy can be exchanged. What is the total entropy of the combined system?

For Problem 2a, we expected the participants to establish the extensive character of entropy (i.e., that doubling the size of the system implies doubling the entropy). A microscopic approach, using the expression $S = k_B \ln W$and multiplication of the number of microstates of isolated systems to calculate the number of microstates of the combined systems, was expected to be a more useful approach for solving the problem than macroscopic ways of reasoning or reference to entropy as disorder. Regarding Problem 2b, we expected the qualitative intuitive answer that the entropy of the systems would be higher when the systems could exchange both particles and energy in comparison to Problem 2a, in which the individual systems were isolated. Problem 2b is quantitatively more challenging than Problem 2a but surprisingly yields the same result, that the entropy is doubled.

Problem 3 (see Figure 3 for diagram) dealt with the adiabatic, reversible expansion of a system:

> Consider a thermally isolated system of an inert gas held in a container by a frictionless piston. Let the gas expand reversibly by moving the piston. What happens to the system's entropy? Does it increase, decrease or remain the same? Justify your answer.

Thermal isolation implies that the process is adiabatic (i.e., that no heat is exchanged with the surroundings). In combination with the expansion being reversible, so that it can come back to the original state, one can calculate the entropy change by $ds = dQ/T = 0J/K$. In other words, from a fundamental macroscopic perspective, one can conclude that the entropy is unchanged in the process of expansion described. However, this conclusion is difficult to draw using a microscopic approach or the disorder metaphor for entropy. As mentioned, empirical studies (e.g., Brosseau & Viard, 1992; Haglund & Jeppsson, in press) have found that many students intuitively feel that the entropy should increase because of the increasing volume, ignoring the energy change.

Overall, although often facilitated by the use of mathematical equations, solving these problems also called for qualitative analysis of physical situations. In all cases, arriving at an adequate solution required drawing on the concept of entropy.

## Procedure

The two PhD students were asked to work on the three problems together without interference from the researchers, except for very brief checkups on their progress. The participants were encouraged to cooperate and discuss their responses with each other. Overall, the approach of letting the participants solve problems in pairs can be justified by the need to bring about a kind of situation that was familiar to the participants and conducive to dialogue. In contrast to the asymmetrical nature of the clinical interview (Schoultz, Säljö, & Wyndhamn, 2001), we aimed for establishing a dialogue between the PhD students on equal terms. In addition, working as a pair was preferred over individual think-aloud problem-solving sessions so as to eliminate the need for any prompting for further thinking and elaboration on the part of the researchers. Given that the goal of the study was to examine the role that subtle use of metaphor played in the problem solving, any intervention on the part of researchers would almost inevitably have introduced language-related metaphorical construals, thereby biasing participants' use of implicit metaphorical resources.

Overall, the PhD students were cooperative when working on the problems and maintained focus throughout the session, which lasted approximately 1 hr. The problem-solving exercise was characterized by strong engagement and mutual dialogue; D2 took on a more leading role, whereas D1 followed but also problematized the proposed solutions. After the problem-solving session the students participated in a debriefing session with the first two authors in which they were asked to reflect on their approaches to solving the problems. The PhD students' problem-solving discussion and participation in the debriefing session was all in Swedish.

### Data Collection and Preparation

The problem-solving sessions were audio and video recorded. A verbatim transcript of the audio recording was prepared in Swedish, the language in which the problem-solving session was carried out. The transcript was then entered into MAXQDA software to support the subsequent categorization and analysis (see the next section). The analysis was performed based on the Swedish transcript by the first two authors, both native speakers of Swedish. The excerpts that are included as illustrations in this article were translated into English by the first two authors. Notes on any issues regarding the translation of metaphorical expressions are included in footnotes to the Results section.

### Data Analysis

Chi's (1997) procedure for analyzing qualitative data served as a guide. The transcript of the problem-solving session as a whole was analyzed. First, the transcript for each problem was partitioned into episodes that reflected a self-contained episode of reasoning. This allowed for coherent stretches of reasoning to serve as the basis for analysis of the particular role played by the metaphorical utterances identified.

Second, episodes identified as particularly rich in metaphorical language were selected for an in-depth analysis. Metaphorical expressions were identified as follows. Utterances that were suspected on initial reading of incorporating a metaphorical expression were subjected to MIP (Pragglejaz Group, 2007), which provides explicit criteria for categorizing a word as metaphorical. In MIP, the contextual meaning of each lexical unit (typically a word) is first interpreted in relation to the whole situation. Next it is ascertained whether the lexical unit has a "more basic contemporary meaning in other contexts than the one in the given context" (Pragglejaz Group, 2007, p. 3); more basic meanings are characterized by being, for instance, more concrete and bodily based. Finally, it is decided "whether the contextual meaning contrasts with the basic meaning but can be understood in comparison with it" (p. 3). If these two criteria are met, the lexical unit is treated as metaphorical; otherwise, it is treated as literal. Following Chi's (1997) top-down approach in combination with MIP, we found metaphorical language in the solution of all three problems and in the subsequent debriefing session. Out of all discourse turns used by the individual participants, 48% were coded as involving metaphorical construals. Next an interrater reliability test was performed on the coding of clauses as either metaphorical or literal by the first two authors on a random sample of the transcripts representing 20% of the collected data. Cohen's kappa was calculated as a reliability measure, yielding a proportion of agreement of 0.86 (values above 0.75 are considered "excellent" according to Fleiss and Levin, 1981).

Third, metaphorical expressions were categorized as instances of underlying CMs. CMs previously identified in scientific texts dealing with the concept of entropy and the second law of thermodynamics (Amin et al., in press) served as an initial coding scheme for the categorization of metaphorical expressions. However, new CM categories were created when metaphorical expressions did not fit into previously identified categories. For the same text sample mentioned in the preceding paragraph, the interrater reliability for kinds of CMs identified at the level of individual clauses was calculated. Individual clauses could be categorized as involving more than one CM, and therefore Cohen's kappa could not be used as a reliability measure, as it requires mutually exclusive and globally exhaustive categories. Instead, we calculated Mezzich's kappa, which is designed to incorporate such overlapping categories (Mezzich, Kraemer, Worthington, & Coffman, 1981); the value was 0.57, following "Procedure 1."[2]

Fourth, a resources perspective was adopted on each episode of the transcript incorporating the use of CM. CMs were then treated as a knowledge resource that could potentially contribute to the reasoning manifested in the episode.

[2]Mezzich et al. (1981) developed their method for interrater assessment of multiple diagnoses in psychiatry, but it has not, to our knowledge, been used before in the field of science education research. In nursing research, the approach has been applied to the assessment of the interrater reliability of qualitative interview data (Eccleston, Werneke, Armon, Stephenson, & MacFaul, 2001). Although no rules of thumb have been identified for which values of Mezzich's kappa are adequate, 0.57 from the present study can be compared to 0.43 found in the study by Eccleston et al. (2001), which was rated as a moderate level of agreement.

In addition, analysis of the reasoning episode also sought to identify any p-prims, understood as image-schematic knowledge structures *not marked linguistically* (see the introductory section), or symbolic forms. The episode was then examined for any evidence suggesting that the CMs played a functional role in the reasoning taking place in the episode. No p-prims (as defined previously) were identified, but a number of symbolic forms were evident in the PhD students' solutions. The use of symbolic forms is presented in the Results section only to shed light on how CMs are used. Repeated passes through the transcript and comparison of episodes led to the identification of a range of CMs with distinct uses, sometimes in coordination with symbolic forms.

## RESULTS

In this section, we present the findings of our analysis. In the first subsection we start with a summary of the reasoning of the two PhD students. In the subsequent subsection we first present a survey of the CMs used, indicating the flexibility in construals they provided, and then describe how the CMs contributed to the problem solving using illustrative excerpts from the transcript.

### Overview of the Problem Solving

When addressing Problem 1, the PhD students first tried to establish the depth of reasoning required and whether to use a microscopic molecular approach or a macroscopic thermodynamic approach. Eventually, they appropriately explained the process in Problem 1 at a macroscopic level in terms of energy moving from the water to the surrounding air by the process of heating. Reasoning quantitatively with the expression $dS = dQ/T$, they justified the requirement that the process leads to an overall entropy increase due to the temperature difference between the system and its surroundings (see the Appendix for a detailed solution to Problem 1). Reasoning qualitatively, they interpreted the phenomenon of the freezing water as a thermodynamic system in thermal contact with a heat bath, and in this way they did not focus on the internal structure of the system. In particular, they did not address the release of energy when bindings form in the phase change of water freezing in the system. From another perspective, despite being explicitly asked to account for the process of freezing of water, they were not distracted by the salient increase in the spatial order of the ice.

In Problem 2a, the participants intuitively assumed that the entropy of two isolated systems regarded together is $2S$ (i.e., double that of one of the systems; see the Appendix for a detailed solution to Problem 2a). The PhD students appropriately justified this view by saying that the number of microstates of the systems regarded together is equal to the number of microstates of one system multiplied by the number of microstates of the other. They established the link to the entropy by stating the proportional relation to the logarithm of the number of microstates, as expressed in $S = k_B \ln \Omega$.

The PhD students used a statistical mechanics approach to Problem 2b with combinatorial reasoning by placing $2N$ particles in the system one by one. They formulated a partition function for $2N$ particles based on the canonical ensemble, which applies to the case in which energy can be exchanged with the surroundings. In fact, this was not the case, as the combined system was isolated from the surroundings, but the PhD students nevertheless reached the correct answer, that the entropy was $2S$ also in this case (see the Appendix for a detailed solution to Problem 2b). As expected, they were puzzled by the result that the answer was the same as in 2a; their reasonable intuition was that it should have been higher, and they were relieved in the subsequent debriefing to hear that the difference lay hidden in the fact that microstates in which the energy and particles are shared equally between the two halves are much more common than any other possible microstates.

The PhD students solved Problem 3 by discussing the possible changes to the state in a pressure/volume (PV) diagram and stating that by definition, reversible processes imply constant entropy. In this way, they approached the problem in a macroscopic, principled way (see the Appendix for a detailed solution to Problem 3). However, one of the participants found the answer strange given the volume increase but accepted the conclusion that entropy was constant by reference to the tradeoff between increased configurational entropy and decreasing internal energy due to the system performing work on the surroundings.

## The Use of CMs in Problem Solving

In this subsection, we present our findings regarding how CMs were used in solving the three problems through a detailed analysis of illustrative excerpts from the transcript. First, under Claim 1, we show how the PhD students construed abstract entities in terms of concrete notions such as objects and agents when reasoning both qualitatively and quantitatively in the thermal domain. Second, under Claim 2, we show how several CMs may be coordinated and how they relate to other cognitive resources, primarily symbolic forms (Sherin, 2001). We argue that this coordination supports the alignment of qualitative and quantitative reasoning. Third, under Claim 3, we show how a sequence of abstract reasoning is simplified and experientially grounded through the use of a "narrative" form of discourse.

### Claim 1: In the context of scientific problem solving, abstract entities are construed flexibly in terms of a variety of concrete notions such as object, spatial location, movement, and agent.

We begin the presentation of our findings by surveying the kinds of metaphorical construals identified in the PhD students' solutions to the problems. To provide some context for this survey we present a number of excerpts from the transcript of the problem-solving session to illustrate the CMs identified. We summarize the full range of CMs identified, the problem solutions in which they were used, and illustrative quotes from the transcript in Table 1.[3]

We begin with an excerpt from the PhD students' solution for Problem 2a. After an initial brainstorming, in which they considered which problem-solving approach to use, the PhD students settled for modeling the relations of each microstate of one system to each microstate of the other, with distinguishable particles across the systems. In the excerpts that follow the two PhD students are denoted D1 and D2.

> [150] D2: Well, OK . . . so for each [system], you have . . . state A has a number of . . . or box A has a number of states *W* . . .
> [151] D1: . . . which gives us the entropy . . .
> [152] D2: . . . in box A, as [writes] S equal to . . .
> [153] D1: ln . . .
> [154] D2: ln *W* . . . and then we have box B . . . and also there we have the entropy . . .
> [155] D2: Uhum . . .
> [156] D2: . . . ln *W,* if we have *W* states in it . . . but then if one compares . . . or takes both at the same time, then you have . . . as we said . . . for the first state in box A, we have *W* states in box B . . . and for the second state in box A, we have *W* states in box B . . .

After a number of false starts in line 150 with "you" and "state A" as subjects, D2 settled for "box A has a number of states *W*." This reflects the CM Microstates Are Possessions (In Containers), an application of the Object Event Structure metaphor, because the abstract notion of microstates is metaphorically construed as objects, possessed by and contained in box A. D1 followed up immediately in line 151 with " . . . which gives us the entropy." Although much was left implicit

[3]As mentioned, the problem-solving discussions were carried out in Swedish, and the excerpts presented here were translated into English by the researchers. The issue of translation is particularly delicate in cognitive linguistics, as it assumes a strong relation between conceptions and language. Swedish and English share a Germanic origin, and the grammar and vocabulary are generally not very different. Therefore, for example, prepositions such as *from* and *to* have direct counterparts in the Swedish *från* and *till.* Although the individual languages are sometimes idiosyncratic when it comes to idiomatic expressions and metaphors—for instance, Lakoff and Johnson's (1980) paradigmatic "I'm in love" becomes patently absurd in Swedish—we did not experience such difficulties in the translation of the participants' discussions. We have aimed to convey the meaning of the sentences and, where possible, use corresponding words in the translations. When a linguistic element, such as a spatial preposition presented in the English translation, is discussed as indicating a concrete spatial source domain, there is an equivalent inference to a spatial source domain from a corresponding linguistic element in Swedish.

TABLE 1
Overview of Identified Conceptual Metaphors From the Problem-Solving and Debriefing Exercises

| *Metaphorical Construal Category* | *Example* | *Problem* |
|---|---|---|
| A Theory Is A Sentient Being | *It [statistical thermodynamics]* doesn't *care* so much about why | 1 |
| Object Event Structure metaphor | | |
| A State Function Is An Object | which gives us *the entropy* . . . *in* box A | 1, 2a |
| Change of A State Function Is An Object | the lower the temperature, the bigger *the entropy gain* you get | 1, 3 |
| Change of (Energy) State Is Transfer Of A Possession (Heat) | If *I take heat* from this beaker with water . . . and *move over to* the room | 1 |
| Microstates Are Possessions (In Containers) | For the first state *in* box A, we have $W$ states *in* box B | 2a |
| Location Event Structure metaphor | | |
| Microstates Are Locations | If you have four *accessible* states . . . and then you maybe *put* two particles *in* it . . . | 2b |
| Change Of State Is Movement | Because if you are going to be able to *get back to the same point,* then you can't increase it either | 3 |
| A Function Is A Machine | For a certain delta $Q$, one *gets* something like this | 1, 2a, debriefing |
| Problem Solving Is Walking Along A Path | One suspects an answer . . . and then one works *towards* it . . . | Debriefing |
| Metaphoric use of pronouns | | |
| A Problem-Solver Is A Manipulator Of A System | If *I take* heat from this beaker with water . . . | 1, 2b |
| A Problem-Solver Is An Operator Of A Machine | . . . which [$S = \ln W$] *gives us* the entropy | 1, 2a, debriefing |
| A Problem-Solver Is A System | Because if *you are going to be able to get back to the same point*, then you can't increase it either, right, because then *you won't get back* . . . | 1, 3 |

in this particular contribution, the verb *give* reflects the use of the A Function Is A Machine CM (Lakoff & Núñez, 1997), in which this function/machine provides entropy as an output with the quantity $W$ construed as an object-like input.[4] The entropy as a function of the number of microstates was further established as they formulated and wrote down the expression $S = \ln W$, where $W$ denotes the number of microstates (although omitting Boltzmann's constant, $k_B$). In line 152, D2 continued with " . . . in box A," thereby construing entropy as object-like and contained in box A, reflecting the A State Function Is An Object CM. Some use of pronouns in this excerpt is also metaphorical. The use of *one* in "but then if one compares" refers literally to the problem-solver as the PhD students changed the focus in their reasoning. However, notice the use of *us* in "which gives us the entropy." *Us* here may be seen as assuming the role of the problem-solver literally, but there is potentially a metaphorical reading as the operator of the machine that gets a particular object as output (i.e., reflecting the CM A Problem-Solver Is An Operator Of A Machine). That is, a metaphorical agent is introduced as a receiver of the output generated by the function/machine.

Flexibility in metaphorical construal of the same abstract entity was found across the different problem solutions. As just noted, in Problem 2a, microstates were construed as objects/possessions. In contrast, in their solution to Problem 2b, the PhD students instead used the Microstates Are Locations CM, an application of the generic Location Event Structure CM, reflected in the phrase "all these states are accessible for all particles." Moreover, in Problem 3 the students used another submapping of the Location Event Structure CM, Change Of State Is Movement. As we will see, this was reflected in "because if you are going *to be able to get back to the same point*, then you can't increase it either," referring to the situation of a reversible expansion of a gas. Another example of flexibility in metaphorical construal is in the use of pronouns to construe the problem-solver metaphorically in a variety of ways. We have seen that metaphorical use of pronouns reflected the CM A Problem-Solver Is An Operator Of A Machine. The use of *you* in this last quoted utterance contrasts the CM A Problem-Solver Is A System, in which the problem-solver is identified (or merged) with the system and is construed as an agent moving along the path that metaphorically construes the changes of state undergone by the system.

The PhD students' solution to Problem 1 revealed some additional CMs, including other metaphorical uses of pronouns. After some discussion about the problem-solving approach, D2 recognized that the situation could be modeled as a thermodynamic system in thermal contact with a heat bath at constant temperature, and he drew the left diagram in Figure 4

[4]Seeing functions as machines reflects Sfard's (1991) idea that mathematical entities can be construed as processes in addition to more formal structures.

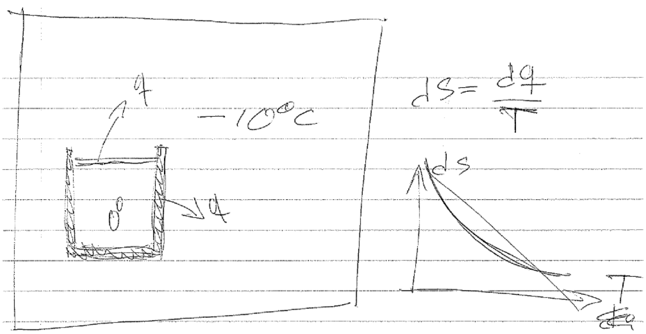


FIGURE 4 Drawing of a thermodynamic system exchanging heat with the surroundings (left). Graph of the change in entropy as a function of the temperature, given an amount of heat added to the system (right).

[57] D2: /. . ./ Well, in this case . . . er, I guess it's simply that . . . if I take heat from this beaker with water . . . and move over to the room . . . in principle, then . . . the partition function in . . . for the room will increase . . . more than what I lose in the beaker, then . . .

In "if I take heat from this beaker with water . . . and move over to the room . . ." D2 spontaneously speaks of heat as if it were an object. This is a metaphor because according to current physics heat is ontologically a process. The metaphor can be expressed as Change of (Energy) State Is Transfer Of Possession. In addition, this utterance includes a metaphorical use of the pronoun *I* reflecting a CM we refer to as A Problem-Solver Is A Manipulator Of A System. Here *I* assumes the role of a manipulator of an aspect of the system, physically moving an object from one place to another. This is a metaphorical use of *I,* as a person cannot literally move heat as an object from one system to another. These two CMs, Change of (Energy) State Is Transfer Of Possession and A Problem-Solver Is A Manipulator Of A System are used together to construe the physical situation that Problem 1 presents. This contributes to the PhD students' qualitative interpretation of the problem. Next, however, in "more than what I lose in the beaker," D2 again makes use of the A Problem-Solver Is A System, identifying with the "loss" in the partition function, and prepares for the quantitative reasoning:

[65] D2: *dS* is *dQ* over *T,* right . . . ?
[66] D1: Delta *S* is equal to delta *Q* over . . .
[67] D2: *T* . . . /. . ./
[75] D2: So, in principle, that means that if the temperature is zero . . .
[76] D1: Uhum . . .

> [77] D2: . . . Kelvin . . . then you *have*[5] an infinite increase in entropy if you *get* a small amount of heat . . . and in principle . . . what I mean with this is that . . . [starts drawing the graph to the right in Figure 4] now, let's see . . . delta *S* as a function of [*T*] for a certain delta *Q*, one *gets* something like this . . . so, maybe . . . the lower the temperature, the bigger the entropy gain you *get* if you move some heat into this system . . . so, that means for our heat bath system here . . . that if you move a small amount of heat from the water beaker out to the room . . . then, the entropy in the room will increase more than the entropy has decreased in the water beaker . . . due to the reason that it is colder in the room . . .

As in their work with Problem 2a, the students use the CM A Function Is A Machine taking objects as input and giving other objects as output, and heat continues to be construed as an object/substance in "if you get a small amount of heat" by use of the CM Change Of State Is Transfer Of Possession. In addition, there are a number of more subtle instances of metaphor reflected in nominalization of processes—that is, cases of grammatical metaphor where processes are talked about using nouns instead of verbs, which would be the typical form in everyday language (see Halliday & Martin, 1993, on grammatical metaphor and Langacker's, 1987, account of the grammatical category noun as associated with a "thing" construal). Examples are "an increase in entropy" and "the entropy gain." Interpreted as a CM, nominalization of increasing entropy is a case of Change Of A State Function Is An Object. This is represented algebraically by the symbol *"dS,"* where the *d* or "delta" is understood as an incremental addition to the existing quantity of entropy. Although entropy itself was conceptualized as an object in Problem 2a by use of a microscopic approach, in this new macroscopic context the entropy increase is treated as an object.

We have mentioned three distinct metaphorical uses of pronouns in which the problem-solver is construed metaphorically: A Problem-Solver Is A Manipulator Of A System, A Problem-Solver Is A System, and A Problem-Solver Is The Operator Of A Machine. Another related construal of the problem-solving process is a CM we have referred to as Problem Solving Is Moving Along A Path. This was reflected in an utterance produced in the debriefing session: "One suspects an answer . . . and then one works *towards* it." In addition, a scientific theory itself was construed as a sentient being in "it [statistical thermodynamics] doesn't care so much about why . . ."

So far we have simply illustrated the kinds of CMs identified in the problem-solving transcript. Our analysis has been limited to pointing out linguistic indicators of metaphorical construals and placing these construals in CM categories. We end this section with an initial indication that CMs are implicated in the reasoning. We point out a situation in which, although the students solved a

[5]Some words in the excerpt are italicized to help in referring to key parts of the excerpt in the analysis.

problem correctly, they made an explicit comment that reflects the operation of a CM that had the potential to mislead en route toward problem solution. In their solution to Problem 3 the PhD students reasoned correctly and from a principled, macroscopic point of view that for reversible processes, the entropy of the system and its surroundings has to be constant. However, as seen here, D1 found this result counterintuitive and "strange" for this particular thermodynamic process:

> [507] D1: It's always strange to think that [the entropy] is the same . . . but, well . . . I guess that's what it is . . . it [the problem-solving approach] goes straight to the entropy . . . that it would be presupposed that one gets more locations to be in . . .
> [508] D2: Uhum . . . what I think happens there . . . is that you . . . if you get enthalpy losses, that turn into entropy . . .
> [509] D1: Uhum . . . that you, well . . . lose in energy, there . . . then you get the entropy . . . well, it . . . one maybe gains on larger volume, but one loses on not having accessible . . . all the energy states . . .
> [510] D2: Uhum . . .
> [511] D1: And hence . . . the entropy is unchanged . . .

With a focus on the volume of the system, D1's intuition was that as the volume increases and therefore the number of possible locations increases, the entropy should increase. Hypothetically speaking, by use of the common construal of Microstates Are Locations, microstates of a system would be regarded as physical locations. In this case, the literal physical volume of the system increases, but the total energy of the system to be distributed among energy levels decreases. Because the energy levels are mapped onto locations only metaphorically, the literally increasing volume comes to the foreground in the reasoning process. The counterintuitive result forces the participants to scrutinize the change in the system also from a microscopic point of view. Eventually, the PhD students' confidence in the belief that the entropy does not change, as deduced from a macroscopic quantitative perspective, constrained their reasoning and allowed them to overcome the apparent "strangeness" caused by their intuition. They identified that the volume increase, giving a positive entropy contribution, is coupled with a loss of internal energy or enthalpy, which reduces the entropy and makes possible the conclusion that the entropy *can* remain unchanged; it is no longer unreasonable from a microscopic point of view. Of particular interest here is that the explicit comment about a conclusion seeming "strange" draws attention to the awkwardness of applying a particular CM. This gives an initial indication that the phenomenon of metaphor in scientific problem solving is not just a superficial linguistic phenomenon. We turn now to a characterization of how CMs and other resources are used together to contribute to reasoning in a problem-solving session.

**Claim 2: Coordination of several CMs and other cognitive resources aligns qualitative and quantitative reasoning.** In this section we show how several CMs may be integrated into a larger coherent schema at each of the qualitative and quantitative levels of reasoning and may serve to align the reasoning at the two levels. In addition, we show how CMs complement the use of other cognitive resources, namely symbolic forms (Sherin, 2001).

Going back to D2's qualitative reasoning about Problem 1 in line 57, we see that in "if I take heat from this beaker with water . . . and move over to the room . . ." there are two CMs involved. First, heat is construed as an object by use of the CM Change Of (Energy) State Is Transfer Of Possession. Second, the CM A Problem-Solver Is A Manipulator Of A System enables D2 to conceptualize *I* as a manipulator that physically moves an object from one place to another. When we consider the source domains of the CMs Change Of (Energy) State Is Transfer Of A Possession (Heat) and A Problem-Solver Is A Manipulator Of A System, we notice that the two source domains combine to form a larger coherent integrated schema—that is, the two source domains fit together. One CM construes objects that can be manipulated, and the other construes a manipulator of objects. The effect of coordinating these two CMs then is to create a single larger compounded CM that could be called Change Of (Energy) State Is Transfer Of A Possession (Heat) By The Problem-Solver used to construe qualitatively the physical situation under consideration.

An even more elaborate coordination of CMs is found in the PhD students' quantitative reasoning in this same problem, as seen in lines 65–77. The CM A Function Is A Machine seems to be a central construal that shapes the conceptualization of physical quantities. The function/machine takes objects as input and gives out objects as output. The function in question is $dS = dQ/T$. Through metaphor the input (heat) and the output (entropy) are construed as objects. Moreover, the PhD students used pronouns metaphorically to introduce a metaphorical agent operating the function/machine providing input and receiving output, as in "you have an infinite increase in entropy if you get a small amount of heat." We suggested earlier that the use of *you* here reflects the CM A Problem-Solver Is An Operator of A Machine at the quantitative level, where "you" receive heat, construed as an object, which leads to "you" having "an infinite increase in entropy." As a whole, four CMs were coordinated by the PhD students as they reasoned quantitatively to arrive at a solution to Problem 1. This is illustrated in Figure 5, in which the source domains of the four different CMs fit together in a coherent way as parts of a large schema: An object/possession (heat) is inputted into a machine (function) by an operator (problem-solver), who receives an object (entropy increase) as output.

We have now seen how the PhD students first made use of two CMs in their qualitative construal of the physical situation under consideration in Problem 1 and then four CMs in their quantitative reasoning. What does all this

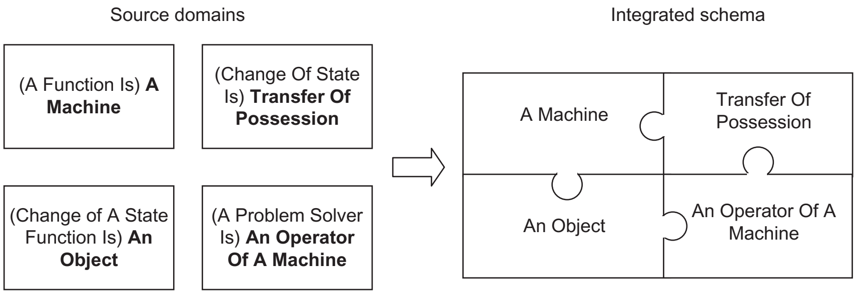


FIGURE 5 Formation of an integrated schema to serve as a source domain for a compound conceptual metaphor used in quantitative reasoning.



use of metaphor achieve? We suggest that these metaphorical construals at the qualitative and quantitative levels establish alignment between the entities in the physical system and quantities manipulated mathematically. One central feature of this alignment is the construal of heat as an object/possession: At the level of the physical system, heat is construed as the movement of an object from one part of the system to another, thus the heat that is "lost" by one part of the system is "gained" by another. At the level of mathematical reasoning, the "lost" heat is construed as input to the function/machine $dS = dQ/T$, applied to the water at 0 °C, providing as output an amount of product that metaphorically construes entropy decrease. Similarly, heat "gained" is construed as input to the function/machine $dS = dQ/T$, applied to the surrounding air, providing as output an amount that metaphorically construes entropy increase. Because temperature is the denominator of $dQ/T$, and the temperature of the surroundings is lower than the temperature of the water in the glass, the output from the machine in the case of heat loss is smaller than the output in the case of heat gain. The conclusion drawn, therefore, is that there is an overall total increase in the entropy of the system and its surroundings that drives the process forward. At the heart of this line of (metaphorical) reasoning linking the qualitative and quantitative levels is the need to interpret the quantity $dQ$ in the two applications of the function as representing identical (yet opposite) changes in the internal energy of the freezing water and the surrounding air. This interpretation is supported by aligned metaphorical construals at each level: At the level of the physical situation, the change in energy is construed as transfer of a possession/heat (therefore, the value of the energy change of each is the same because, in the transfer of a possession, what is gained is identical to what is lost); at the level of mathematical reasoning, $dQ$ is understood as representing heat construed as "an amount" input into the function/machine. Thus, the object/possession construals of heat at both levels enable interpretation at both levels of reasoning. Alignment (see Figure 6) between the metaphorical construal of the qualitative

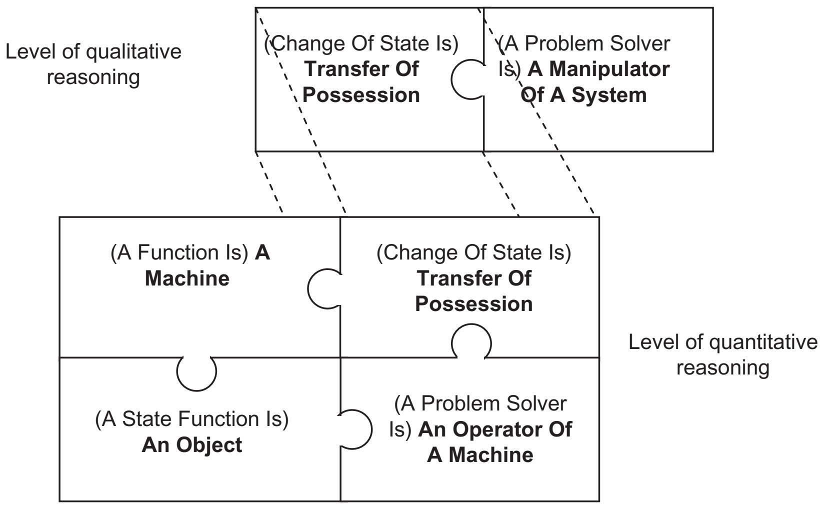


FIGURE 6 Alignment of the metaphorical construals of the physical process and the construals making up the quantitative modeling, with both sharing the conceptual metaphor Change Of State Is Transfer Of Possession, in which heat is seen as an object.

and the quantitative reasoning can be seen as another form of coherence in the coordinated use of CMs, somewhat different from the coherence involved when an integrated schema emerges from the combined use of more than one CM in one single line of reasoning. The construal of heat as an object is shared among the coordinate metaphorical construals at the two levels and so creates a bridge between reasoning at the two levels. However, although the source domains at each level combine to create coherent integrated schemas, there is no suggestion that these schemas at the two levels are merged. We see below a case in which this kind of merging between metaphorical construals at qualitative and quantitative levels can occur.

In the preceding analysis we have focused on the coordinations of CMs only and their contribution to the reasoning. Yet our examination of the problem-solving session also took into consideration the potential role of other cognitive resources such as symbolic forms (Sherin, 2001). The analysis of the PhD students' reasoning in Problem 1 revealed the use of three symbolic forms in addition to the CMs just discussed. First, in line 77, there is a use of the "prop-" symbolic form, which is made up of the symbol template $\left[\frac{\ldots}{\ldots x \ldots}\right]$ and the conceptual schema "as the denominator increases the term as a whole decreases." Prop- is used to interpret the expression $dQ/T$ with a particular focus on the variation of the temperature, the denominator of the expression. In this way, it is crucial to the reasoning that the lower the temperature, the greater the entropy change.

Second, $dQ/T$ provides information about the incremental change in entropy. This implicates the symbolic form "base ± change," "□ ± Δ," within the "terms are amounts" cluster. Here a base term, the initial entropy, is changed by adding a term interpreted as "gain" (in which heat has been added to the surrounding air) or "loss" (in which heat has been lost from the water in the beaker). Finally, the use of the "terms are amounts" cluster enabled the PhD students to compare the values of the entropy gain and loss, respectively, as if they were two objects of different "size." In our view, this way of comparing two entities resembles the symbolic form "opposition," "□ – □," although this symbolic form belongs to the "competing terms cluster," which was first described by Sherin (2001) in relation to influences and directed quantities such as opposing forces. We propose that the comparison of the gain and loss of entropy the PhD students performed reflects another symbolic form that could be added to Sherin's "terms are amounts" cluster, which might be called "different amounts" with the symbol pattern [□ – □]. The overall conclusion drawn, that the increase in entropy in the surrounding air is more than the decrease in entropy in the water, relies on the use of these three symbolic forms.

We believe that these symbolic forms are crucial to the reasoning performed by the PhD students in this problem. However, we believe that the CMs are additional resources that contribute in an important way to the reasoning process. First, as already seen, overlapping construal of heat as an object at the qualitative and quantitative levels aligns the reasoning at these two levels. Although symbolic forms provide meaningful interpretations of elements of equations and clearly contribute to linking the qualitative and quantitative levels, Sherin (2001) has not elaborated on the details of that link in terms of cognitive resources.[6] Second, using the prop- symbolic form in coordination with both the "base and change" and "different amounts" symbolic forms requires interpreting the result of dividing $dQ$ by $T$ as an object/amount. This is achieved by the Function Is A Machine CM. Indeed, both "base and change" and "different amounts" symbolic forms are part of the "terms are amounts" cluster in Sherin's (2001) account. The idea of "terms are amounts" is a metaphorical notion and relies on a metaphorical mapping between abstract mathematical quantities and concrete amounts of a substance. Metaphorically construing heat as an object/substance (input to a machine) and construing entropy as an object/substance (output from the machine) provides the metaphorical construals needed to link the relevant scientific quantities with the symbolic forms. In sum, we hypothesize that the use of CMs links qualitative and quantitative reasoning and provides the needed construal of physical quantities that supports the application of symbolic forms to the quantitative reasoning.

[6]Here the principal ambiguous meaning of a physical quantity (such as heat) as denoting both a quality and a quantity (Strömdahl, 2012) is at stake. This is metaphorically solved by reifying heat as an object overcoming the "qualitative/quantitative bridge" facilitating the problem-solving process.

Claim 3: Grounding abstract reasoning in metaphorical construals allows the problem-solver to draw on the rich inferential resources provided by experience-based intuitive knowledge structures realized linguistically in the basic narrative form. We have seen how multiple metaphorical construals can be coordinated to align qualitative and quantitative reasoning. In this subsection we would like to extend the argument for the contribution of CMs to scientific problem solving by clarifying the role of the source domains of CMs understood as experience-based knowledge structures. We suggest that the grounding of abstract reasoning in metaphorical construals provides an experiential/narrative base rich in inferential structure, thus making the reasoning more transparent, almost intuitively obvious. This is best illustrated in the dramatic case of construing the problem-solver in such a way as to endow him or her with characteristics of the system itself, as seen in the solution to Problem 3. Here, the PhD students reasoned about an expanding system by referring to a PV graph representing how the pressure varies with the volume. In this case, coherent coordination of multiple CMs goes so far as to blend[7] the problem-solver, the system, and the graphical representation of the physical quantities. After reading Problem 3, a task dealing with the adiabatic, reversible expansion of a system, the PhD students turned straight to the fundamental character of a reversible process:

> [488] D2: So, the definition of a *reversible* process actually is that the entropy does not change . . .
> [489] D1: Well, right . . . [draws a PV diagram] It is that . . . it's a question of that one *walks along the same line* . . . if one increases the volume . . . and then, when one decreases the volume, then . . .
> [490] D2: . . . you can *get back to* the same state . . .
> [491] D1: Yes, right.
> [492] D2: Then you can't have had any entropy losses . . . because you can never decrease the entropy in an isolated system . . .
> [493] D1: No.
> [494] D2: Because if you are going to be able to *get back to the same point,* then you can't increase it either, right, because then you won't *get back* . . .

First, consider the italicized words and phrases: *reversible, walks along the same line, get back to the same point,* and *get back to.* All of these reflect a sustained use of the Change Of State Is Movement CM. Second, the use of the pronouns

[7]Use of the word *blend* can be seen as similar to the more technical use of the term as used in the conceptual integration framework of Fauconnier and Turner (2002) and applied by Hutchins (2005) in considering how mental and material input spaces are combined and how understanding and reasoning emerges when "running the blend." Although we believe that a conceptual integration analysis can be given to this and other examples we discuss in this article, we are not using the word *blend* in this technical sense here.

*one* and *you* in phrases like "one walks along the same line" and "you won't get back" reflects a metaphorical construal of the problem-solver as identified with the system. Instead of saying that the system goes from some state to another, the speaker construes the problem-solver as doing the moving (i.e., reflecting the CM A Problem-Solver Is A System). Moreover, the line on the PV graphical representation is construed as the path along which the problem-solver/system travels. Thus, the physical system, the problem-solver, and the graphical representation are superimposed or blended into one coherent construal of an individual walking along a path. This is the same kind of "indeterminate reference" identified by Ochs et al. (1996) and reviewed in the introductory section. Although Ochs et al. suggested that this intermediate form of discourse between physicist- and physics-centered discourse seems to be associated with a "thinking through process," they did not characterize the nature of the contribution this form of discourse makes to scientists' reasoning per se. We put forward a hypothesis here regarding the role that this form of discourse plays in the PhD students' problem solving.

We have already made the contrast between a coherent integrated schema used for either the qualitative or quantitative construals of the system independently. We have also seen coherent coordination of CMs aligning qualitative and quantitative reasoning. The case of blending just presented goes beyond just alignment in that it seems to create a coherent concrete schema that allows for the qualitative and quantitative reasoning to be seamlessly integrated, not just aligned. The use of the verb *walks* reflects this concreteness: The problem-solver (identified with the system) is construed as a human being, moving as human beings do, along a path (used to construe the changes of state undergone by the system).

The image of an individual walking along a path able to reverse directions to return at will and without obstacle to the starting location is part of the justification offered by the PhD students for why the entropy does not change in this situation of a reversible expansion of a gas that is thermally isolated from its surroundings. What cognitive role might this image be playing? By using the CM Change Of State Is Movement, the students construe the states of the system as locations and changes as movements between locations. The assumption of reversibility requires that the magnitude of changes from one state to another be infinitesimally small. Metaphorically speaking, this is achieved by construing the movement of the system along a line, as lines are made of infinitely many adjacent points that are infinitesimally close together. Moreover, as D1 emphasized, it is movement along "the same line." This is important because it guarantees that all of the points (i.e., states of the system) implicated in the movement are all parts of a continuous path of connected points, hence infinitesimally close to one another. It could be suggested that reference to a continuous curve on a PV graph supports the construal of the movement between points as taking place on a continuous path. That is, the visual graphical representation supports the metaphorical construal expressed linguistically. Although the graphical representation gives greater concreteness and something to refer to, the metaphorical construal involved is dynamic. That is, the elaboration of this image in terms of the problem-solver walking along the line adds the notion of unobstructed agency, as found in "it's a question of walking along the same line" and "able to get back to the same point." This provides the image of a freely moving entity able to access all points on a continuous path and thus further reinforces the notion of reversibility.

Although this analysis gives a sense of how the metaphor works, it does not tell us why it is so useful to use. One way to capture the value of the metaphorical construal is to imagine what would be involved in trying to avoid the metaphor when talking and thinking about processes that involve state change, and in particular reversible processes. An attempt to construe this process as literally as possible would result in something like the following: "First, at the initial time $t_1$, the system is of state $S_0$. Then, later, because of changes of the system, at time $t_2$, it is of state $S_1$. If the system undergoes a number of changes, so that there are many consecutive states that are very similar to each other, it is possible for the system at some later time $t_3$, to be of the same state as it was at $t_{11}$. We could then conclude about this set of consecutive states produced in this way that the entropy of the system is the same for all points in the time specified." This is indeed a very awkward series of sentences, but that is the point: We have avoided expressions such as *in state* $S_0$, *originally, go back to, reverse, step, move, walk,* and *path,* which all involve construing temporal change in terms of movement in space. (But note that we did not manage to get rid of the spatial interpretation of time in "at the initial time $t_1$"; it is virtually impossible to avoid metaphor entirely.)

The ideas and reasoning presented in this set of (almost) non-metaphorical sentences are far from transparent. When one grounds these abstract ideas in the concrete notion "of walking along a line," all of the crucial features of reasoning fall out intuitively: Changes of state can be as small as one wants them to be, because lines are continuous collections of points, and initial states can be readily accessed simply by reversing direction of movement. One way to describe this metaphorical construal is to say that it transforms abstract reasoning into a narrative involving an agent engaged in a journey in which he or she seeks to arrive at a certain destination by moving along a path that is smooth, with no obstacles. The coordinated use of CMs creates a narrative form reflecting a central aspect of human experience, and when one draws on concrete notions of space and movement, the path is understood intuitively and readily generates inferences. Moreover, this metaphorical construal greatly simplifies the reasoning that would require many steps in a chain of reasoning were metaphor to be avoided (as seen in the non-metaphorical paraphrase above).

## DISCUSSION

In the Results section, we presented the outcome of our analysis of the PhD students' problem-solving session in the form of three claims: First, we showed that the PhD students construed abstract theoretical concepts metaphorically in terms of objects, locations, possessions, paths, movement, transfer of possessions, and agents in a flexible way, in the context of both qualitative and quantitative lines of reasoning (i.e., they made extensive use of CM). Second, we suggested that by coherently coordinating several CMs, as well as symbolic forms, the PhD students aligned their qualitative and quantitative reasoning. Third, we argued that their coordinated use of CM could be seen as the adoption of a "narrative" form of discourse that simplified complex abstract reasoning, grounding it in inferentially rich experiential knowledge structures.

In this discussion, we interpret these findings in light of the three themes emphasized in previous research, as reviewed in the introductory section, and highlight the contribution of the present study in each case. First, we discuss how CM relates to the issue of how sense-making resources from everyday thinking can contribute to expertise in science. Second, we elaborate on the perspective of understanding as embodied, with a particular emphasis on the recent debate regarding the ontological classification of concepts by the novice and expert. Third, we discuss how external representations of physical phenomena by use of semiotic systems, such as language, visual representations, and algebraic equations, relate to internal cognitive resources. In our discussion of each theme we point out the implications of our findings for understanding the process of learning. We end the discussion by pointing out implications of this research for education.

### Theme 1: CM as a Cognitive Resource

As mentioned in the introductory section, prior research has identified a variety of knowledge elements and processes that are used in everyday sense making but that could also potentially contribute to understanding of, and reasoning about, phenomena in line with current science. Previous research has identified p-prims, symbolic forms, anchoring intuitions, imagistic simulation, analogical reasoning, and others. In illustrating the role that the coordination of CMs plays in aligning qualitative and quantitative reasoning and simplifying complex abstract reasoning by grounding it in experiential knowledge, we suggest here that CMs can be added to this list of everyday sense-making resources that can contribute to scientific understanding and reasoning. It was argued in the introductory section that the source domains of CMs, the conceptual schemas of symbolic forms and p-prims, can all be seen as subsets of the larger set of image schemas. What is distinct about CMs is that they involve mappings across domains, recruiting readily available sense-making resources in one domain to construe abstract entities and processes in another. The pervasive use of metaphor in the problem-solving session analyzed reveals the importance of mapping between domains. We further highlight this importance by noting that some symbolic forms (Sherin, 2001, 2006) generally not seen as involving metaphor do indeed involve a mapping between domains. For example, the "terms are amounts" cluster is based on a metaphorical interpretation of numbers as if they were objects, as recognized by Lakoff and Núñez (2000).

Although the CMs are reflected in aspects of language, we view the evidence provided regarding the role that the source domains of the CMs play in the problem solving as indicating that we are not dealing with a superficial linguistic phenomenon but one in which conceptual schemata are triggered by linguistic elements. If the concrete source domains reflected in the language used were not conceptually significant we would expect the use of metaphor to be unsystematic, varying randomly from one metaphorical usage to another. This is not what we find on close scrutiny of the metaphorical construals used in particular contexts of problem solving. We also suggest that CMs complement other resources for problem solving, particularly symbolic forms.

For instance, Sherin (2001) argued that symbolic forms are utilized in the meaningful interpretation of algebraic formulas, thereby supporting the coordination of quantitative and qualitative levels of reasoning. We have suggested that CM plays an important role in enabling such coordination. We illustrated this through the central role played by the CM A Function Is A Machine. We argued that this CM supported the coordination of the symbolic form *prop-* with the qualitative reasoning in Problem 1. Moreover, we noted that this was achieved via the coordination between A Function Is A Machine and other CMs. As we argued in Claim 1, physical quantities tended to be construed as objects, but it is in relation to the A Function Is A Machine CM that we start to understand why this happens. We suggest that the use of one CM has a strong influence on which other CMs will be drawn upon in the reasoning and how this will be done. The PhD students' successful problem solving involved object-like construals of heat and change in entropy in a way that cohered with the input/output construal requirements of the CM A Function Is A Machine. It was in the context of the use of multiple, coherently coordinated CMs that the symbolic form was used. We suggest that this kind of coordination contributes to what diSessa and colleagues (diSessa, 2002; diSessa & Sherin, 1998; diSessa & Wagner, 2005) have referred to as coordination classes. In their view, a characteristic of expert reasoning is the ability to coordinate many diverse knowledge elements in a particular context in a way that is sanctioned in science. In turn, learning is viewed as the gradual incorporation of useful knowledge elements into an integrated system of elements characterized by appropriate cueing priorities. What is distinctive about the coordination of CMs is that once a quantity, qualitative entity, or process is construed metaphorically in some way (e.g., an as input or output substance) it is likely that other construals consistent with it will be encouraged (e.g., construing heat and entropy change as object-like). One might say that one metaphorical construal will afford other metaphorical construals.[8] This suggests the possibility that an emergent process of coordination of CMs takes place during the learning process. Further research is needed to provide further empirical support for this hypothesis.

In conclusion, we have suggested that in scientific problem solving multiple CMs are coordinated with one another and with symbolic forms, allowing for quantitative and qualitative reasoning to be aligned and thereby coordinated. We therefore propose that CMs should be added to the list of intuitive, non-formal knowledge structures that have been found to support scientific understanding and reasoning, not as an alternative resource but rather as one complementary to others proposed in the literature.

### Theme 2: CM Grounds Abstract Reasoning in Experiential Knowledge

We have provided initial evidence that the source domains of the CMs identified contributed to the PhD students' reasoning used to solve the problems. The use of CM was pervasive in the problem-solving session analyzed. Because the source domains of the CMs used consisted of image-schematic structures, the PhD students construed abstract entities, such as states and changes of state, in terms of more concrete entities, such as objects, agents, spatial location, and movement. That is, the CMs used grounded abstract concepts and reasoning in experiential knowledge supporting a rich array of inferences. This finding lends support to the position of diSessa, Gupta, and colleagues in the ongoing debate regarding novices' and expert scientists' ontological categorization of concepts (Gupta et al., 2010; Hammer et al., 2011; Slotta, 2011; Slotta & Chi, 2006). As mentioned earlier, diSessa (1993b) and Gupta et al. (2010) challenged the view put forward by Chi and colleagues (e.g., Slotta & Chi, 2006) that novice and expert understanding can be contrasted in terms of a stable ontological classification of concepts, with the former categorizing many scientific concepts as material substances and the latter categorizing these same concepts more abstractly as constraint-based (or emergent) processes. Gupta et al. (2010) argued that both learners and experts construe concepts within ontological categories in a considerably more flexible way, with experts often construing abstract concepts in terms of more concrete notions of material substance. In response to this critique, Slotta (2011) acknowledged that according to his and Chi's position, experts may hold parallel ontologies for a concept under exceptional circumstances, such as wave-particle duality in modern physics. However, he argued that observed cases in which scientists may

[8]We acknowledge the contribution of the reviewers of an earlier version of this article to our formulation of the coordination of metaphors in this way.

classify an abstract concept within more than one ontological category are often not scientifically adequate. He accepted that scientists will continue to use a more concrete ontology in everyday, and possibly some pedagogical, contexts but will not treat this as scientifically acceptable. To illustrate, Slotta (2011, p. 157) stated that "experts can still think of electric current as 'juice' squirting through a wire but would quickly acknowledge that such a substance does not actually exist."

This debate may derive in part from differing assumptions and points of emphasis. Whereas the ontological shift perspective seems to insist on explicit ontological classification judgments on the part of scientists (as Slotta's, 2011, preceding example suggests), the resources perspective has been more interested in identifying intuitive, often implicit resources that can contribute to scientific understanding and reasoning. It is not necessary that scientists explicitly accept the claims of resource theorists (i.e., the concrete, experiential basis of what they take to be abstract [often formal] conceptual understanding). Regardless of how a scientist might respond to the explicit ontological question "What kind of thing is X?" the researchers investigating implicit cognitive processes may still identify concrete, experiential notions playing a role in the scientist's understanding and reasoning. Indeed, when diSessa (1993a, p. 274) wanted to show that certain concepts are difficult to fit into *any* of the ontological categories suggested by Chi and colleagues, he used entropy as an example: "Entropy itself does not belong to any obvious naive ontology. It is not material, or a process, or a mental state." Despite this difficulty in explicit ontological classification, it is still possible to claim that scientists will sometimes construe entropy and changes in entropy in terms of various concrete experiential conceptualizations while reasoning through problems. In particular, through the use of CMs, our two PhD students frequently construed entropy and other abstract thermodynamic states and processes in terms of a material substance ontology. In a quantitative microscopic context, by using the function $S = k_B \ln \Omega$, the PhD students talked about entropy, a state function, as if it were an object that can reside within a system. In a macroscopic context, entropy was construed qualitatively as a position on a vertical scale, in relation to heat, which was given a substance-like interpretation. Quantitatively speaking, however, using the function $dS = dQ/T$, the students construed the entropy *gain* as an object, an output from the function/machine. Given their adequate grasp of thermodynamics, we do not doubt that they would reject any explicit suggestion that entropy, a state function, is a material substance. But they do put the construal implicitly to use when needed.

Apart from construing physical quantities as concrete objects, the PhD students also made extensive use of CMs that involved different degrees of identification of the problem-solver with the system: We saw the problem-solver construed as manipulating the system and even *being* the system. For example, in "if I take heat from this beaker with water and move over to the room," the PhD students introduced themselves as an active agent interacting with the system. And in their solution to Problem 3, they conflated the role of the problem-solver with the system itself, as seen in "one walks along the same line." Wilensky and Reisman (2006) have documented how identification with an individual entity in a complex system (e.g., a wolf in an ecosystem) can support learning about aggregate-level phenomena and their quantitative description. In the metaphorical construals we report here, we see that the cognitive benefits of identification can extend beyond cases of identifying with actual agents in a complex living system. Here nonliving physical entities and processes are construed anthropomorphically. Moreover, the complex relationships between physical entities and processes and their graphical and quantitative representation are blended along with the problem-solver, reducing the complexity of abstract reasoning required to relate levels to a single concrete image of an agent moving along a path. As mentioned, the issue of relating levels of representation of physical phenomena to one another is important in learning thermodynamics (Baierlein, 1994; Slotta & Chi, 2006), in reasoning about the concept of entropy, as well as in many other domains of science. Thus, the extent to which implicit use of metaphor is important in grounding abstract multilevel reasoning is worth pursuing in future research on expertise in thermodynamics and other domains of science.

In his response to Gupta et al. (2010), Slotta (2011, p. 153) hypothesized, "Perhaps there is a linguistic propensity in how people talk about such [scientific] concepts that would bias children toward such materialistic interpretations." Slotta was commenting on children's conceptions, but our findings and those of others suggest that such a propensity is found in scientists as well. This is the phenomenon of CM! We have argued that the source domains of CMs consisting of experience-based schemata are recruited in advanced scientific problem solving, casting further doubt on the claim that concept learning involves a shift from one stable ontological category to another. Scientific reasoning draws on experience-based conceptions of objects as well as agents to ground that reasoning in schematized experience with rich inferential structure.

Thus far we have discussed CM as a cognitive phenomenon. However, as we have pointed out throughout this article, and as Slotta (2011) pointed out in the preceding quote and Gupta et al. (2010) discussed, this phenomenon is closely tied to language. We turn next to discussing the significance of the connection between CM and language viewed as one of a number of representational tools.

## Theme 3: Metaphor, Language, and Other Semiotic Systems

In this section we widen the perspective and discuss the issue of the interaction between cognitive resources and a range of semiotic systems. We highlight the flexibility of language-based construals and in particular how linguistic resources are recruited to create a narrative form of discourse.

Although our discussion thus far has emphasized the nature of the metaphorical construals used by the PhD students and their contribution to problem solving, it is important to recognize that these construals are marked in language. For instance, drawing on the resources of CMs implicit in everyday language, the PhD students selectively used the Location Event Structure metaphor or the Object Event Structure metaphor at the service of local contexts of reasoning. Shifting between these two types of CMs involves subtle figure-ground reversals (e.g., from microstates construed as locations in certain circumstances to microstates construed as objects in others). Such shifts in construal are a hallmark of language use, richly explored by cognitive linguists (Lakoff, 1987; Lakoff & Johnson, 1999; Langacker, 1987; Talmy, 2000). Moreover, it has been argued that subtle shifts in language-based construals are implicated in cognitive development (Amin, 2009; Budwig, 2003; Tomasello, 1999). We noted earlier that Svensson et al. (2009) observed great flexibility in novice students' use of language to express their intuitions about physical phenomena. This flexibility reflected incoherence and inconsistency in understanding and thinking and was seen as reflecting a lack of expertise. However, from a cognitive linguistic perspective, the flexibility in the PhD students' construals in the present study was a systematic and coherent phenomenon, revealing consistency and coordination throughout a particular line of reasoning but highly flexible across different problems and levels of reasoning, both qualitative and quantitative, microscopic and macroscopic.

Moreover, the PhD students coordinated their language-based construals with other semiotic resources. The language-based construals contextualized the use of symbolic forms and supported bridging between qualitative and quantitative levels of description. In addition, language-based construals elaborated the interpretation of a graphical representation in the PV diagram. In both cases of coordination between the language-based construals and another representational system, the outcome was narrative-like, coherent accounts of manipulated objects, moving along paths, input into machines or received as output, changing in size or amount, and so on. This was made possible by the range of available construal options associated with linguistic elements. Thus, abstract formal reasoning using graphs and equations was realized as a form of narrative discourse.

Bruner (1991) has contrasted scientific, paradigmatic discourse to the more accessible and familiar narrative form. As noted earlier, however, Ochs et al. (1996) suggested that when scientists are thinking through problems together, their discourse is characterized by identification and active interference with the studied systems that encompasses features from narrative discourse but is still part of professional language. A feature of narrative discourse mentioned by Bruner is that in narratives, events are structured in time. Although not an example of CM, the use of cognitive/linguistic resources to construe simultaneous processes as if they were events ordered in time is another example of the flexible use of language to impose subtle construal shifts for a variety purposes. In Problem 1, the structure in time can be illustrated by "the entropy in the room will increase more than the entropy has decreased in the water beaker." Here, two processes that actually occur simultaneously at the same level of the physical system are talked about as if they occur at distinct moments in time, as seen in the use of the past tense in "has decreased" and the future tense in "will increase," which may be interpreted as a case of *grammatical* rather than *conceptual* metaphor (Halliday & Martin, 1993). Apart from structuring events in time at one level of reasoning, narration is also involved in the alignment of different levels of reasoning. For instance, "If I take heat from this beaker with water and move over to the room . . . then the partition function . . . will . . ." relates the qualitative construal of the process causally by the "If . . . , then . . ." statement to the quantitative formulas by grammatical separation in time. In addition, by the introduction of an agent in the quote, the PhD students made use of the narrative techniques of bringing up a particular example in making a more general point and introducing a human being with intentions, interpreted in terms of causality. In our view, the examples show how the competent PhD students made productive use of narratives in their problem solving, which provides a contrast to diSessa's (1998) account of novices' superficial use of narratives.

In this section, we are highlighting the link between the metaphorical construals discussed in this article and the linguistic forms that express them. Although we have emphasized the role of the metaphorical mapping per se as being of particular cognitive interest, recruiting experience-based source domains in reasoning the question of the significance of the link to language must be raised. It is in principle possible for all of the experience-based resources discussed to be invoked independently of language. However, the conventional association between metaphorical mappings and linguistic forms motivates the speculation that the linguistic expression of these mappings may play an important role in guiding attention and perspective shifting (see Amin, 2012, for discussion). Tomasello (1999) has argued that language learning makes an important contribution to the richness of perspectival shifts emerging over the course of cognitive development. The significance of the cognitive contribution of linguistic expression per se associated with metaphorical mappings in the context of scientific problem solving needs to be examined empirically in further work, most likely requiring an experimental research design.

Seeing the use of CM and potentially also other resources as linked to language is also significant because of the communicative function and the possibilities for learning that arise. Discursive and sociocultural approaches to learning science highlight participation in communities of practice as an important contributor to learning (J. S. Brown, Collins, & Duguid, 1989; Yerrick & Roth, 2005). The emphasis in our discussion of Themes 1 and 2 has been the contributions made to scientific understanding and reasoning by the source domains of CMs. But seeing these CMs as intimately tied to elements of language suggests that exposure to and participation in linguistic (and more generally discursive) practices provides pointers to experiential cognitive resources that can support abstract understanding and reasoning. Future research needs to examine the process of appropriating the scientifically sanctioned use of these resources through participation in discursive practices.

## Implications for Education

In line with the assumption of continuity in the reasoning of novices and experts, we assume that the source domains of the CMs identified as contributing to scientific problem solving in this study are available to the novice learner. However, as the research on science learning from a resources or knowledge-in-pieces perspective has taught (diSessa, 1993b; Hammer et al., 2005; Smith, diSessa, & Roschelle, 1993), the availability of resources does not guarantee their correct application and coordination. In the present study, even these well-trained PhD students were close to being misled by overinterpreting the CM Microstates Are Locations, a phenomenon recognized by Brookes and Etkina (2007). By commenting that "it's strange" that the entropy can remain constant at adiabatic reversible expansion, the students gave emphasis to the spatial contribution to entropy but implicitly neglected the energy contribution, which would require a metaphorical interpretation. As pointed out in previous research on conceptions of thermal phenomena among undergraduate students, ignoring the energy contribution often leads to the wrong conclusion that the entropy increases in such reversible adiabatic expansion (Brosseau & Viard, 1992; Haglund & Jeppsson, in press). As Brookes (2006, p. 13) has pointed out, however, language offers few opportunities to speak about abstract ideas, such as states of a system, in a literal way, and metaphorical ways of talking are unavoidable. A problem that arises is that novices tend to interpret such metaphorical expressions literally. For instance, a scientist can use the spatial metaphor "the electron is in the ground state" to easily generate valid inferences, but a novice may interpret this literally as the electron being at a location in space. Moreover, seeing heat as an object that is moved from one location to another may easily lead to the conclusion that it is a conserved quantity, the refutation of which led to the abandonment of the caloric theory in the mid-19th century. Similarly, the idea of entropy as an object does not fit well with the fact that it is an increasing quantity. However, this does not mean that CMs cannot be involved in construing entropy as ever increasing; in line with the findings of Williams (2012) with regard to learning to read the clock, it is a matter of which CMs to use and how to apply them in a particular context.

Brookes (2006) suggested that teachers use language that is consistent with the correct ontological categories of scientific concepts. For instance, in relation to the topic of the present study, he proposed the use of “Energy flows from A to B by heating” as an improvement over the typical “bad sentence” of “Heat flows from A to B.” However, as discussed previously, our findings and those of others adopting a resources perspective suggest that this kind of pedagogical language “purity” not only is unrealistic but also is not consistent with the nature of scientific expertise. In more recent writing, Brookes and Etkina (2007, 2009) have increasingly acknowledged the flexibility of scientific conceptualization and language use. In this way, they now treat language-based conceptualizations as tools used with contextual sensitivity within scientific practices, in line with our findings. Rather than try to avoid the use of metaphor, an alternative approach is to discuss potentially misleading interpretations explicitly with students and work on developing a metalinguistic awareness of the metaphorical features of the language of science.

In addition to helping learners avoid misinterpretations of CMs implicit in scientific language, educators can find needed entry points for instruction in an understanding of how scientists use CMs in reasoning and problem solving. For example, in this article we have described how PhD students used multiple CMs to align their qualitative and quantitative reasoning. Preliminary findings from further research we have just initiated with two undergraduate chemistry students working together with the same problems as the PhD students reported here suggest that these undergraduate students made considerably less use of CM than the PhD students in their reasoning, particularly when it came to the metaphorical use of pronouns to construe themselves (as problem-solvers) in interaction with or identified with the system. Their language was more formal and reflected greater distance between themselves and the phenomena under consideration. Although they were generally successful in arriving at an adequate solution, a characteristic of their problem solving was that they tended to start by trying to remember formulas that involved physical quantities that seemed relevant to the task (i.e., following the typical backward reasoning of the novice; e.g., Larkin et al., 1980). In contrast, as we have seen, the PhD students in the present study started by framing the problem qualitatively and establishing an appropriate quantitative approach, hence by use of the expert forward reasoning (e.g., Larkin et al., 1980). One tempting conclusion is that in general the PhD students have appropriated strategies of metaphorical construal that allow for seamless transition from the physical situation being considered to the use of algebraic equations to draw quantitative conclusions.

If it is correct that part of expertise in problem solving involves strategic metaphorical construal to align qualitative and quantitative aspects of reasoning, this might be explicitly targeted in instruction. For example, the finding that a number of CMs are coordinated in the context of scientific problem solving may be considered in conjunction with the proposal to use multiple analogies in science teaching (Aubusson, Harrison, & Richie, 2006; Duit, 1991; Spiro, Feltovich, Coulson, & Anderson, 1989).[9] It is possible that identifying implicit CMs in scientific thinking can guide the design and selection of multiple explicit instructional analogies. That is, CMs suggest the design of explicit instructional analogies to incorporate the same conceptual mappings to support alignment and articulation of levels of reasoning. However, such a suggestion must remain tentative for how implicit CMs relate to explicit instructional analogies requires further theoretical and empirical investigation (see Amin et al., in press, for an initial examination of this issue).

In conclusion, we have provided support for the view that one can construe physics concepts and processes in a flexible, context-dependent way in problem solving by referring to concrete objects and agents through the use of CMs. Moreover, we have suggested ways in which these concrete notions might contribute to problem solving in science. However, the investigation of how such use of CM contributes to expertise and is appropriated by learners is just beginning. Much research still needs to be done to characterize the learning process and its implications for instruction.

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[9]Although we have made the point that concrete knowledge gestalts are readily drawn on to reason about entropy, we would not advocate the instructional tactic of treating entropy exclusively as a substance-like quantity, as has been done within the Karlsruhe physics approach (Fuchs, 1987; Herrmann, 2000). From our perspective, that approach amounts to selecting a single CM and constructing a single, extended instructional analogy around it.

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# APPENDIX
## Detailed Solutions for the Problems in the Problem-Solving Session

*1. A beaker contains water at temperature 0 °C and is put in a room of air at a constant temperature of –10 °C, so that a layer of ice forms on top of the liquid water. Describe what drives forward the process of freezing the water.*

This problem can be modeled at the macroscopic level as the beaker representing a thermodynamic system in thermal contact with its surroundings, such that any transfer of water molecules between the beaker and the environment is neglected. When water freezes to ice, more hydrogen bonds are formed than when water is in the liquid phase; in this process energy is released, some of which is transferred to the surroundings in the form of heat: *Q*. Because there is less energy to be distributed in the system, the entropy of the system decreases. In addition, due to the fact that the mixture of water and ice is in equilibrium during freezing, the process is reversible, and one can calculate the entropy change of the system: $\Delta S_{sys} = -Q/T_{sys}$, where $T_{sys} = 0 + 273K = 273K$ is the constant temperature of the system in Kelvin during the freezing. However, the received heat in the surroundings gives a positive contribution to the entropy: $\Delta S_{surr} = Q/T_{surr}$. Because the surrounding temperature is lower than that of the system, the total entropy change is positive: $\Delta S = \Delta_{surr} = \Delta S_{sys} = Q/T_{surr} - Q/T_{sys} > 0J/K$. This increase in entropy drives forward the exchange of heat and the process of freezing.

*2. Two identical isolated systems A and B contain a monatomic ideal gas, each one with number of particles N, volume V, temperature T and entropy S.*

*(a) When the two systems are considered together as 'A and B,' the temperature is T, the volume is 2V and the number of particles is 2N. What is the entropy of the combined system 'A and B'?*

Entropy is an extensive variable, and therefore it increases with the increased size of the system, a characteristic that is shared with the volume *V*, the internal energy *U*, and the number of particles *N*. This extensive character can be seen by using a statistical mechanics approach and the laws of logarithms. Here entropy is introduced as a function of the number of microstates $\Omega$ in $S = k_B \ln(\Omega)$, where $k_B$ is Boltzmann's constant. Because systems A and B are identical, in isolation they both have the same number of microstates $\Omega_A = \Omega_B$ and therefore also the same entropy $S_A = S_B$. Each microstate of one system can be related to $\Omega$ microstates of the other system, so the number of microstates is a multiplicative quantity. Hence, taken together, but still isolated from each other, the two systems have the entropy $S_{tot} = k_B \ln(\Omega_A \Omega_B) = k_B \ln(\Omega_A^2) = 2k_B \ln\Omega_A = 2S_A$.

*(b) Assume that the two systems are put in contact, so that particles and energy can be exchanged. What is the total entropy of the combined system?*

Problem 2b can be related to Problem 2a, and we show that the entropy of the systems is $2S$ also in this case. The unintuitive conclusion that the results for Problems 2a and 2b are the same stems from the fact that the number of configurations in which the energy and particles are distributed evenly between the two halves is *much* higher than the number of any other possible configurations. Specifically, if a certain number of microstates are available to the systems taken together, but in isolation from each other, then those microstates are still available in this new situation. In addition, there will be a large number of new microstates relating to the situation in which particles and energy have been exchanged between the systems, but, as we show, this contribution can be neglected because of its small size in relation to the original number of microstates. Consequently, the entropy of the systems as described in 2b will be larger than the result in 2a, $2S_A$, but only marginally so. Hypothetically speaking, if there are $\Omega$ microstates in one macroscopic system and $a\Omega$ microstates in another macroscopic system (i.e., the number of microstates is increased by a factor of $a$, which may be large but is still *very much* smaller than $\Omega$), the entropy of the first system is $S_1 = k_B \ln \Omega$ and the entropy of the other system is $S_2 = k_B \ln a\Omega = k_B \ln a + k_B \ln \Omega$. Because $\ln a$ is *very much* smaller than $\ln \Omega$, one can safely use $S_2 \approx S_1$ as an extremely accurate approximation. Quantitatively speaking, one approach to solving the problem is to make use of the Sackur-Tetrode formula, in which the additional energy contributions apart from the most probable ones are ignored: $S = Nk_B \left\{ \ln\left(\frac{V}{N}\right) + \frac{3}{2}\ln\left(\frac{2\pi m k_B T}{h}\right) + \frac{5}{2} \right\}$, where $N$ is the number of particles, $k_B$ is Boltzmann's constant, $V$ is the volume, $m$ the mass of a particle, $T$ is the temperature, and $h$ is Planck's constant. When one considers the two systems put together so that they can exchange particles and energy, $N$ and $V$ are doubled in comparison to one of the initial systems but the other variables remain constant. Therefore, the expression within the brackets is constant and the entropy of the total system is $S = Nk_B\{\ldots\} = 2S_A$. Alternatively, if one is modeling the system as a canonical ensemble (which, however, requires the possibility of exchanging heat with the surroundings, as opposed to the current isolated system), the same expression could be deduced from the partition function of $N$ indistinguishable particles: $Q = \frac{q^N}{N!}$.

*3. Consider a thermally isolated system of an inert gas held in a container by a frictionless piston. Let the gas expand reversibly by moving the piston. What happens to the system's entropy? Does it increase, decrease or remain the same? Justify your answer.*

In thermodynamics a change is said to be reversible if it "can go in the other way" to restore the original state of the system and its surroundings. In ideal circumstances, this can be achieved by carrying out changes to the system infinitely slowly and keeping the system infinitesimally close to equilibrium at all times.

In the current problem, an inert gas (i.e., a gas in which no chemical reactions are assumed to occur) is kept in a container by a frictionless piston. Now, the gas expands adiabatically (i.e., no heat $Q$ is exchanged with the environment) and reversibly by moving out the piston. When the piston is moved out, work $W$ is performed by the gas on the environment, which means that the internal energy $U$ and temperature $T$ of the gas decrease. At the same time, the volume $V$ increases. The entropy does not change in the process because no heat has been exchanged with the environment and the process is reversible, such that the following relation holds: $dS = \delta Q/T = 0J/K$. If approached from a microscopic perspective, the increasing volume makes a positive contribution to the number of microstates and therefore to the entropy, but there is also a negative contribution to the entropy due to the reduced internal energy. Because we have argued macroscopically that the total entropy of the system does not change, these two contributions have to cancel out entirely, but this is far from easy to deduce using a purely microscopic approach.